Low price signals high capacity*

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Abstract

We study pricing in a model where buyers are homogeneous and sellers have either capacity one or two. We show that if buyers observe prices but not capacities then an equilibrium exists where sellers of capacity two post lower prices than sellers of capacity one. The equilibrium satisfies the intuitive criterion.

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1 Introduction

The size of a firm and the wages the firm offers have been shown to be positively related in empirical research (see, e.g., Brown and Medoff, 1989, Winter-Ebmer and Zweimüller, 1999). Most theoretical models of directed search, however, predict just the opposite.

The positive size-wage differential, which refers to large firms posting higher wages, cannot be obtained in models where the only dimension of heterogeneity is the number of vacancies. This is because the workers are interested both in the wage and the probability of getting a job. Large firms offer a higher probability of getting a job than small firms and in equilibrium they are able to capitalise this in the form of low wages.

Theoretical models that feature equilibria consistent with the empirical findings assume asymmetries among the firms. Lester (2010) constructs a labour market model with directed search where firms can employ either one or two workers, costs of employing are convex, and all firms have the same productivity per worker. In the benchmark setting, large firms post lower wages because they can offer higher probability of employment. He extends the model by assuming that productivity differs across firms. The more productive firms are large and offer higher wages in equilibrium. Tan (2010) arrives at a similar result by introducing an optimal level of hiring capacity for firms. When firms that produce optimally with two employees fill only one or no positions, they have an incentive to post high wages so as to attract more job candidates and subsequently produce more per worker.

The situation is analogous in the goods market where the empirical finding is that large firms charge lower prices than small firms (Roberts and Supina 1996), while the theoretical models predict the opposite. For instance, Burdett, Shi and Wright (2001) show that sellers of capacity two post higher prices than sellers of capacity one. Godenhielm and Kultti (2012) endogenise the capacity of the sellers in a goods market model and demonstrate that equilibrium prices differ for sellers if their capacities differ, with a larger seller asking a higher price. Shi (2002) studies sequentially active labour and goods markets, where firms can produce with two employees at most, and shows that large firms charge higher prices and pay their workers higher wages than small firms. The large firms earn higher revenue per employee because they offer potential buyers a higher probability of trade, and thus are able to pay their employees more. Hence, the model in Shi (2002) explains the positive size-wage differential in the labour market, but the prices of large firms are still high.

The common assumption in the above articles is that prices and wages as well as firm size are observable. We give up this assumption and set up a model of buyers and sellers, where the buyers do not observe the size of the sellers, and consequently have to base their contact strategies on observed prices only. This may depict a situation where the sellers cannot credibly communicate their capacities. One may also think of this as an exercise where the objective is to see what can be communicated by price alone. We show that there exists a multitude of separating equilibria in which the high-capacity sellers post lower prices than the low-capacity sellers. We also find the unique equilibrium that satisfies the intuitive criterion.

In many real life settings, it is impossible for a job candidate to assess how many vacancies a firm is trying to fill and for a buyer to determine how many units of a product a seller has. Hence, this information has to be inferred from the posted price. In our setting price signals the capacity, and posting a low price is a costly and credible signal that a seller offers many goods. We find this result interesting but mention two caveats. First, we assume that the sellers cannot credibly communicate capacity whereas they can communicate and commit to prices. While this may be a reasonable description of goods markets where the sellers are bound to advertised prices by law, and monitoring the capacity is difficult, it is perhaps not such a good description of markets for, say, expensive durable goods. Second, we do not endogenise the decision to have high or low capacity; some firms are assumed to possess one unit for sale and others two units. Allowing capacity to be a decision variable is far from trivial (cf. Godenhielm and Kultti, 2012).

A central function of pricing in our model is to communicate private information. Menzio (2007) presents a search model where firms are privately informed on the quality of jobs they offer. They cannot credibly communicate this nor commit to compensation but there exists an equilibrium in which cheap talk by firms leads workers to contact the high-quality firms more often than low-quality firms. As the former pay more, the end result is reminiscent of the standard directed search model. In our model cheap talk is not possible but firms can completely commit to prices.

Under non-observable capacity the sellers with high capacity do worse than under observable capacity. This hints that it is in their interest to find ways to communicate their capacities to the buyers. Perhaps for this reason it is difficult to find real life examples that are very close to our theoretical model. But the model is still useful in understanding the incentives the sellers have as to pricing.

The article is organised as follows. In the next section we outline the model which is standard except for the assumptions what the agents observe. The third section contains the main results and analysis. In the fourth section we present some concluding remarks.

2 The model

In the economy there are S sellers and B buyers. A seller is referred to as "he" and a buyer as "she". Denote the ratio of buyers to sellers by $\theta \equiv \frac{B}{S}$. A fraction y of the sellers has two units of an indivisible good for sale, and the rest of the sellers, $(1-y)S$, have one unit of the good for sale. The sellers with one unit are called low-capacity sellers, and the sellers with two units are called high-capacity sellers.

The model is static, the sellers maximise expected profits, and the sellers' reservation value of the goods is zero. Buyers are homogeneous, each buyer has a unit demand and values the good at unity. The sellers post prices, p_1 and p_2, where the subscript stands for the capacity, at which they commit to sell a unit of the good. The buyers observe the prices, but not the capacities, and contact the sellers based on the price information.

As is typical in directed search models we focus on a symmetric equilibrium which means that the buyers use mixed contact strategies. This results in a meeting process that can be viewed as random. It also captures the market frictions as the sellers may meet any number of buyers. Consequently, locally there may be under supply and over supply.

For large S and B we can approximate the number of buyers a seller expects to meet by a Poisson-distribution. We denote the Poisson-parameter for a high-capacity seller by $\alpha \equiv \frac{x}{y}\theta$ and for a low-capacity seller by $\beta \equiv \frac{1-x}{1-y}\theta$, where x denotes the fraction of buyers that contact sellers with capacity two. The Poisson-parameters give the expected number of buyers per a high-capacity and a low-capacity seller, respectively. In the sequel we refer to them as (expected) queue lengths.

The key variable is x. The sellers can affect it by their pricing; if, say, high-capacity sellers lower their prices then more buyers contact them, or x grows, which means that fewer buyers contact the low-capacity sellers. In the end it is determined by the buyers' optimal behaviour; they must be indifferent between contacting different sellers.

The probability that a low-capacity seller meets exactly k buyers is $\frac{\beta^k}{k!}e^{-\beta}$, and he can sell his sole item only if he meets one or more buyers. His expected profit is

$$\pi_1 = (1 - e^{-\beta})p_1 \tag{1}$$

A high-capacity seller sells one unit if he is contacted by exactly one buyer and two units if two or more buyers visit him. The probability that he is visited by one buyer is $\alpha e^{-\alpha}$, and by more than one buyer $1 - e^{-\alpha} - \alpha e^{-\alpha}$ and his expected profit is

$$\pi_2 = \alpha e^{-\alpha} p_2 + 2(1 - e^{-\alpha} - \alpha e^{-\alpha})p_2 \tag{2}$$

If a seller is contacted by more buyers than he has capacity, each buyer has an equal probability of obtaining a unit of a good. Her realised utility from trade is the valuation minus the price, $1 - p_i$, $i \in \{1, 2\}$. A buyer's expected utility from visiting a low-capacity seller is

$$U_1 = (1 - p_1)\frac{1 - e^{-\beta}}{\beta} \tag{3}$$

The first factor on the RHS gives the surplus from obtaining the good at price p_1. The second factor gives the probability that the buyer obtains the good; this is the probability that at least one buyer contacts the seller divided by the expected number of buyers at the seller, β.

A buyer's expected utility from visiting a high-capacity seller is

$$U_2 = (1 - p_2)\frac{\alpha e^{-\alpha} + 2(1 - e^{-\alpha} - \alpha e^{-\alpha})}{\alpha} \tag{4}$$

The interpretation of (4) is analogous to that of (3), the most important difference being that if the seller meets more than one buyer then two units of a good are allocated between the buyers.

If a seller posts price $p \notin \{p_1, p_2\}$ then the expected number of buyers who contact him depends on the buyers' belief whether the seller has low or high capacity. Assume that the deviator is believed to have low-capacity. Then the buyers' optimal behaviour makes them indifferent between the sellers, and the expected queue length γ must satisfy

$$(1 - p)\frac{1 - e^{-\gamma}}{\gamma} = U_1 = U_2 \tag{5}$$

3 Equilibrium

We focus on perfect Bayesian separating equilibrium. In equilibrium the sellers' pricing strategy is a best response to other sellers' pricing strategies and the buyers' contact strategies, and the buyers' contact strategy is a best response to the sellers' pricing strategies and other buyers' contact strategies. The buyers have expectations about the capacity of the sellers that are based on prices and the expectations are confirmed in equilibrium. We construct a separating equilibrium where the prices are such that $p_1 > p_2$, buyers use symmetric contact strategies, and the equilibrium beliefs are that sellers who post off-equilibrium prices have low capacity.

Definition 1 *A symmetric separating equilibrium with B buyers, yS high-capacity sellers and $(1-y)S$ low-capacity sellers is given by the prices p_1, p_2, the expected profits of the sellers π_1, π_2, the expected utilities of the buyers U_1, U_2 and the buyers' belief that price p_1 is by low-capacity sellers, price p_2 is by high-capacity sellers and any price $p \notin \{p_1, p_2\}$ is by a low-capacity seller, such that*

1. $U_1 = U_2$,
2. *Given the buyers' beliefs and that x adjusts so that 1. holds the prices p_1 and p_2 maximise π_1 and π_2,*
3. *The expectations are rational,*
4. *The buyers' response to an off-equilibrium price $p \notin \{p_1, p_2\}$ is given by expression (5).*

Notice that in the definition we consider only deviations $p \notin \{p_1, p_2\}$ that still attract some buyers; if a deviator's price is sufficiently unfavourable no buyers contact him.

Proposition 1 *There exists a continuum of symmetric separating equilibria with low-capacity sellers' price* p_1 *and high-capacity sellers' price* p_2*, such that* $p_1 > p_2$*. The buyers' out of equilibrium beliefs are that any price* $p \notin \{p_1, p_2\}$ *is by a low-capacity seller. In the best equilibrium for the high-capacity sellers the prices are* $p_1 = \frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\beta}}$ *and* $\overline{p}_2 = \frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\alpha}}$*, and the queue lengths are determined by* $(1-p_1)\frac{1-e^{-\beta}}{\beta} = (1-\overline{p}_2)\frac{\alpha e^{-\alpha}+2(1-e^{-\alpha}-\alpha e^{-\alpha})}{\alpha}$*.*

Proof. In Appendix A.1. ■

In the sequel we focus on the best equilibrium for the high-capacity sellers. Below we briefly explain the intuition behind the result while a detailed proof is relegated to the appendix.

In equilibrium the expected utility of a buyer must be the same whether she contacts a high-capacity seller or a low-capacity seller. Setting (3) equal to (4) we obtain the buyers' indifference condition

$$(1-p_1)\frac{1-e^{-\beta}}{\beta} = (1-p_2)\frac{\alpha e^{-\alpha} + 2(1-e^{-\alpha}-\alpha e^{-\alpha})}{\alpha} \equiv \bar{U} \tag{6}$$

which determines the magnitudes of α and β. Note that if $p_2 \leq p_1$, then the buyers' indifference condition implies that $\alpha > \beta$.

The price of the high-capacity sellers, p_2, must be such that the low-capacity sellers do not find it profitable to ask p_2 or

$$(1-e^{-\alpha})p_2 \leq (1-e^{-\beta})p_1 \tag{7}$$

The LHS of (7) gives the expected profit for a low-capacity seller if he chooses the large sellers' equilibrium price inducing demand given by the queue length α. On the RHS is the expected profit of a low-capacity seller at price p_1, as in (1).

Denote by $\overline{p}_2$ the price that makes the low-capacity sellers just indifferent. Among all separating equilibria the one with prices p_1 and $\overline{p}_2$ generates the highest profits to the high-capacity sellers.

As the low-capacity sellers are constrained only by the fact that they have to offer the market utility $\bar{U}$ to the buyers the equilibrium p_1 is gotten in the standard way (see appendix); the explicit expression is given by

$$p_1 = \frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\beta}} \tag{8}$$

Notice that price competition is mitigated by the capacity constraints. Lowering price a little does not result in discrete increases in the expected number of buyers as the buyers value also the probability of getting a good. The price, or the share that the seller gets, is the probability that there is competition for the good, i.e., two or more buyers, divided by the probability that there is at least one buyer.

Using (8) we can solve for $\overline{p}_2$ when (7) holds as an equality

$$\overline{p}_2 = \frac{1 - e^{-\beta} - \beta e^{-\beta}}{1 - e^{-\alpha}} \tag{9}$$

Given this p_2, it is clear that the low-capacity sellers do not want to deviate to it, and neither to any other price because p_1 maximises π_1 subject to offering the market utility $\bar{U}$ in (6). The high-capacity sellers do not want to post a higher price because buyers expect a deviator to be of low capacity, which means a discrete decrease in the queue length. A higher price would be detrimental for the same reason.

It is clear that there exists a continuum of separating equilibria with $p_2 < \overline{p}_2$ supported by the unfavourable beliefs that deviators have low capacity.

3.1 Intuitive criterion: Separating equilibria

We argue that the constructed separating equilibrium has appeal since it satisfies the intuitive criterion. The formal definition can be found eg. in Cho and Kreps (1987) but since there are just two types it is straightforward to describe this refinement. An equilibrium does not satisfy the intuitive criterion if there is some type that could profitably deviate given the most favourable beliefs from the deviator's point of view, while no other type would find the deviation profitable under any beliefs. In the present setting it is clear that the deviators to consider are the high-capacity sellers. The most favourable belief for them is that they are expected to have high capacity.

Assume that in a separating equilibrium a deviation would be profitable to a high-capacity seller if he were believed to be a high-capacity seller. The intuitive criterion is satisfied if this same deviation under the same beliefs would be profitable to the low-capacity sellers, too. In this case no-one could argue that the buyers' equilibrium belief, that any deviator has low capacity, is unreasonable.

When the buyers' belief is that deviations to price $p \notin \{p_1, \overline{p}_2\}$ are by high-capacity sellers the indifference conditions are given by

$$\frac{1 - e^{-\beta}}{\beta}(1 - p_1) = e^{-\beta} = \frac{2 - 2e^{-\alpha} - \alpha e^{-\alpha}}{\alpha}(1 - \overline{p}_2) = \frac{2 - 2e^{-\gamma} - \gamma e^{-\gamma}}{\gamma}(1 - p) \tag{10}$$

We have two cases to cover.

Lemma 1 *If a deviation by a high-capacity seller to price $p > \overline{p}_2$ is profitable, it is profitable to a low-capacity seller, too.*

Proof. Let $\Omega_z \equiv 2 - 2e^{-z} - ze^{-z}$. Notice that if $p > \overline{p}_2$, then (10) implies that $\gamma < \alpha$. Notice also that if $p > 1 - \frac{\Omega_\alpha}{\alpha}(1 - \overline{p}_2)$ then the deviator does not attract even a single buyer. From (10) we can solve $p = 1 - \frac{\gamma}{\Omega_\gamma} e^{-\beta}$. As the deviation is profitable, we have

$$\Omega_\gamma p = \Omega_\gamma - \gamma e^{-\beta} > \Omega_\alpha \frac{1 - e^{-\beta} - \beta e^{-\beta}}{1 - e^{-\alpha}}. \quad (11)$$

At this price a low-capacity seller would make $(1-e^{-\gamma}) - (1-e^{-\gamma})\frac{\gamma}{\Omega_\gamma} e^{-\beta}$ and we want to show that this is greater than the equilibrium profit or $1 - e^{-\beta} - \beta e^{-\beta}$. Let us instead show a stricter relation that

$$(1 - e^{-\gamma}) - (1 - e^{-\gamma})\frac{\gamma}{\Omega_\gamma} e^{-\beta} > \frac{1 - e^{-\alpha}}{\Omega_\alpha}\left(\Omega_\gamma - \gamma e^{-\beta}\right),$$

where we have used (11). This is equivalent to

$$\frac{1 - e^{-\gamma}}{\Omega_\gamma} > \frac{1 - e^{-\alpha}}{\Omega_\alpha},$$

which certainly holds as $\gamma < \alpha$. ■

Lemma 2 *Assume that a high-capacity seller deviates to price $p < \overline{p}_2$. It is not profitable to start with.*

Proof. Assume to the contrary that the deviation is a profitable or

$$\Omega_\gamma p > \Omega_\alpha \overline{p}_2.$$

The buyers' indifference condition between the deviator and the high-capacity sellers who ask $\overline{p}_2$ is given by

$$\frac{\Omega_\gamma}{\gamma}(1 - p) = \frac{\Omega_\alpha}{\alpha}(1 - \overline{p}_2).$$

Dividing sideways the above expressions and manipulating a little one gets

$$p > \frac{\alpha F}{\gamma G + \alpha F}, \quad (12)$$

where $F \equiv 1 - e^{-\beta} - \beta e^{-\beta}$ and $G \equiv e^{-\beta} + \beta e^{-\beta} - e^{-\alpha}$. Next we use the buyers' indifference condition between the deviator and the low-capacity sellers, $\frac{\Omega_\gamma}{\gamma}(1 - p) = e^{-\beta}$, to solve

$$p = 1 - \frac{\gamma}{\Omega_\gamma} e^{-\beta}.$$

We claim that

$$1 - \frac{\gamma}{\Omega_\gamma} e^{-\beta} < \frac{\alpha F}{\gamma G + \alpha F}, \quad (13)$$

which contradicts (12). Expression (13) is equivalent to

$$\Omega_\gamma < e^{-\beta} \frac{\gamma G + \alpha F}{G}.$$

Notice first that both sides of the inequality are increasing in γ. The smallest possible value of γ is $\gamma = \alpha$ as the deviation is downwards. Substituting from (10) $e^{-\beta} = \frac{\Omega_\alpha}{\alpha}\frac{1-e^{-\alpha}-(1-e^{-\beta}-\beta e^{-\beta})}{1-e^{-\alpha}}$ into the inequality and evaluating it at $\gamma = \alpha$ one finds that both sides are equal. The derivative of the LHS is $e^{-\gamma}(1+\gamma)$, an expression decreasing in γ, and the derivative of the RHS is constant $e^{-\beta}$. Consequently, if we can show that $e^{-\alpha}(1+\alpha) < e^{-\beta}$ we have shown our claim.

From (10) one sees that the greater the discrepancy between the low-capacity price p_1 and the high-capacity price p_2 the greater is the difference between α and β. Let us assume that $p_2 = p_1$. Then the indifference condition becomes

$$\frac{1-e^{-\beta}}{\beta} = \frac{\Omega_\alpha}{\alpha}.$$

We know that $\beta < \alpha$ which means that $1-e^{-\beta} > 2-2e^{-\alpha}-\alpha e^{-\alpha}$. Let us next assume that $e^{-\beta} = e^{-\alpha}(1+\alpha)$, which gives

$$1-e^{-\alpha}-\alpha e^{-\alpha} > 2-2e^{-\alpha}-\alpha e^{-\alpha},$$

which clearly does not hold. Consequently, it must be the case that $e^{-\beta} > e^{-\alpha}(1+\alpha)$.

■

Let us finally note that the separating equilibrium we have constructed is the only one that satisfies the intuitive criterion. If the price asked by the high-capacity sellers p_2 is less than $\overline{p}_2$ then no deviation $\widetilde{p}_2$ such that $\widetilde{p}_2 \in (p_2, \overline{p}_2)$ is profitable to the low-capacity sellers by construction. For high-capacity sellers at least small deviations are profitable given that the buyers believe the deviator to be a high-capacity seller. This can be seen by totally differentiating the buyers' indifference condition

$$e^{-\beta} = \frac{\Omega_\gamma}{\gamma}(1-\widetilde{p}_2) \tag{14}$$

to get

$$\frac{\partial\gamma}{\partial\widetilde{p}_2} = \frac{\Omega_\gamma}{e^{-\gamma}(1+\gamma)(1-\widetilde{p}_2)-e^{-\beta}}.$$

The objective function of a deviating seller is $\Omega_\gamma\widetilde{p}_2$ and the derivative evaluated at $\widetilde{p}_2 = p_2$ (resulting in $\gamma = \alpha$) turns out

$$\Omega_\alpha - \frac{2\left(1-e^{-\alpha}-\alpha e^{-\alpha}-\frac{\alpha^2}{2}e^{-\alpha}\right)}{\alpha^2}\frac{\Omega_\alpha}{e^{-\alpha}(1+\alpha)(1-p_2)-e^{-\beta}}p_2$$

This is certainly greater than zero if $e^{-\alpha}(1+\alpha)(1-p_2)-e^{-\beta} < 0$. Substituting $e^{-\beta}$ from (14) this is equivalent with $2\left(1-e^{-\alpha}-\alpha e^{-\alpha}-\frac{\alpha^2}{2}e^{-\alpha}\right) > 0$ which holds for all values of α.

We can thus state

Proposition 2 *The separating equilibrium with prices* $p_1 = \frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\beta}}$ *and* $\overline{p}_2 = \frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\alpha}}$ *is the unique one that satisfies the intuitive criterion.*

3.2 Intuitive criterion: Pooling equilibria

Here we show that there are no pooling equilibria which satisfy the intuitive criterion. In this sense the only interesting equilibrium is the separating one we focus on above. We leave it open whether pooling equilibria exist in the first place, and just show that if there is a pooling equilibrium then it fails the intuitive criterion. The idea of the proof is that a high-capacity seller can post a price low enough such that the low-capacity seller does not find it profitable to imitate, while he still finds the deviation profitable because he has more goods for sale.

Proposition 3 *No pooling equilibrium satisfies the intuitive criterion.*

Proof. Assume that there is a pooling equilibrium in which everyone posts price p. A buyer's expected utility is given by

$$(1-p)\left[y\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta}+\frac{1-e^{-\theta}}{\theta}\right].$$

Let the buyers' expectation be that a deviating seller has high capacity. Consider a seller who deviates to price $p-\varepsilon$, where $\varepsilon>0$. The queue length, $\gamma(\varepsilon)$, that he faces is determined by

$$(1-(p-\varepsilon))\frac{\Omega_{\gamma(\varepsilon)}}{\gamma(\varepsilon)}=(1-p)\left[y\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta}+\frac{1-e^{-\theta}}{\theta}\right].$$

Allowing ε approach zero continuity implies that in the limit we have

$$(1-p)\frac{\Omega_{\gamma}}{\gamma}=(1-p)\left[y\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta}+\frac{1-e^{-\theta}}{\theta}\right],$$

where γ is the limit of $\gamma(\varepsilon)$. It is immediate that $\gamma(\varepsilon)>\gamma>\theta$. Consequently, for sufficiently small ε it must hold that

$$\Omega_{\gamma(\varepsilon)}(p-\varepsilon)>\Omega_{\theta}p.$$

It is also clear that there exists ε' such that

$$\Omega_{\gamma(\varepsilon')}(p-\varepsilon')=\Omega_{\theta}p. \tag{15}$$

We show that at this price the low-capacity sellers make strictly less than at price p. First solve from (15)

$$p-\varepsilon'=\frac{\Omega_{\theta}}{\Omega_{\gamma(\varepsilon')}}p,$$

A low-capacity seller's profit at this price is

$$\left(1-e^{-\gamma(\varepsilon')}\right)\frac{\Omega_{\theta}}{\Omega_{\gamma(\varepsilon')}}p.$$

This is less than at price p if

$$\left(1-e^{-\gamma(\varepsilon')}\right)\frac{\Omega_\theta}{\Omega_{\gamma(\varepsilon')}}p < \left(1-e^{-\theta}\right)p,$$

which is equivalent to

$$\frac{\Omega_\theta}{1-e^{-\theta}} < \frac{\Omega_{\gamma(\varepsilon')}}{1-e^{-\gamma(\varepsilon')}},$$

which holds as $\gamma(\varepsilon') > \theta$. This means that a price slightly greater than $p-\varepsilon'$ is a profitable deviation to high-capacity sellers, when the buyers' expectation is that the deviators are high-capacity sellers, while it is not profitable to the low-capacity sellers. ■

3.3 Example

To get some idea how non-observability of the capacity affects the outcome let us consider an economy where there is an infinite number of agents, a unit mass of sellers with capacity one, a unit mass of sellers with capacity two and a mass three of buyers. So we assume that $S=2$, $y=\frac{1}{2}$ and $B=3$. This means that $\alpha = 3x$ and $\beta = 3(1-x)$.[1]

3.3.1 Observable capacity

One can derive the equilibrium prices in the same manner as the price of low-capacity sellers is derived in the appendix. They turn out

$$q_1 = \frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\beta}},$$

and

$$q_2 = \frac{2-2e^{-\alpha}-2\alpha e^{-\alpha}-\alpha^2 e^{-\alpha}}{2-2e^{-\alpha}-\alpha e^{-\alpha}}.$$

The buyers' equilibrium condition determines x, and under observable capacity it is given by

$$\frac{1-e^{-\beta}}{\beta}(1-q_1) = \frac{2-2e^{-\alpha}-\alpha e^{-\alpha}}{\alpha}(1-q_2).$$

We numerically solve $x = 0.686\,35$. This results in prices

$$q_1 = 0.397\,76,$$

$$q_2 = 0.457\,85.$$

[1] Based on simulations this example is typical. The calculations are available on request.

3.3.2 Non-observable capacity

The buyers' equilibrium condition that determines x is given by

$$e^{-\beta} = \frac{2 - 2e^{-\alpha} - \alpha e^{-\alpha}}{\alpha} \times \frac{e^{-\beta} + \beta e^{-\beta} - e^{-\alpha}}{1 - e^{-\alpha}},$$

where the values of α and β, of course, differ from the values in the observable case. We again numerically solve $x = 0.821\,08$, and this results in prices

$$p_1 = 0.244\,49,$$

and

$$\overline{p}_2 = 0.111.$$

3.3.3 Comparison

Both prices are lower when capacity is not observable. This is because the only way to signal capacity is via low prices which increases the competitive pressures for all sellers. Under observable capacity the price of the high-capacity sellers is more than four times the prices under non-observability.

One should notice that, though nice to the buyers, the low prices when capacity is not observable result in a socially suboptimal outcome. Given yS high-capacity sellers and $(1-y)S$ low-capacity sellers the total number of trades given by

$$yS\left(2 - 2e^{-\alpha} - \alpha e^{-\alpha}\right) + (1-y)S\left(1 - e^{-\beta}\right) \tag{16}$$

where $\alpha = \frac{xB}{yS}$ and $\beta = \frac{(1-x)B}{(1-y)S}$. Maximising this expression with respect to x yields the following condition

$$e^{-\alpha}(1+\alpha) = e^{-\beta},$$

but this is exactly the buyers' indifference condition under observable capacity. We know from the analysis of the non-observable capacity that in the equilibrium

$$e^{-\alpha}(1+\alpha) < e^{-\beta},$$

which shows the claim about socially suboptimal outcome.

In our example the welfare measure (16) is $1.017\,6$ under non-observable capacity, and $1.046\,0$ under observable capacity. From the welfare point of view it does not make much difference whether capacity is observable or not; but it determines which party gets the surplus from trade.

4 Concluding remarks

If buyers are worried about stock-out, and if sellers cannot credibly communicate how much they have for sale then price acts as a signal of capacity. We have shown that this leads to pricing that is in accordance with empirical findings. Namely, large sellers post lower prices than small sellers, as low price is a credible and costly signal about capacity. This is in stark contrast to the complete information model where large sellers post higher prices than small sellers as they capitalise on the lower stock-out probability.

An immediate extension would be to allow for sellers with arbitrary unequal capacities. We have not been able to extract the results for such a setting. The reason is that the no-deviation condition for large sellers might cease to hold automatically once the no-deviation condition for small sellers is satisfied. This is because of the interplay between capacities and expected queue lengths which are non-linearly related. Endogenising the capacity choice would be a further step. One could also, in the spirit of Shi (2002), try to determine whether lower prices by large sellers in the goods market requires large sellers to pay their employees lower wages or whether they can simultaneously pay higher wages and price lower than small sellers.

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A Appendix

A.1 Proof of Proposition 1.

The proof is divided into steps. First we establish two technical results which guarantee that certain variables are well-defined, then we derive the equilibrium prices, check for deviations in the unique equilibrium that satisfies the intuitive criterion, and finally we show that there is a continuum of separating equilibria.

Lemma 3 *Whenever $p_2 < p_1$ in a separating equilibrium the queue length for high-capacity sellers is larger than for low-capacity sellers, i.e., $\alpha > \beta$.*

Proof. In the buyers' indifference condition the LHS is decreasing in β and the RHS is decreasing in α. Even when the prices are the same the RHS is greater than the LHS when $\alpha = \beta$. Consequently, equality requires that $\alpha > \beta$. ■

Lemma 4 *The buyers' indifference condition determines a unique correspondence between α and β for $p_2 < p_1$.*

Proof.

We prove that a unique solution exists to (6) such that $\alpha > \beta > 0$. Differentiating the LHS with respect to x it is easily seen to be positive. Differentiating the RHS with respect to x it is easily seen to be negative. When $x = y$ we have $\alpha = \beta = \theta$. Then the LHS is less than the RHS. Increasing x either produces equality for a unique value $x \in (0, 1)$ or the LHS remains less than the RHS even when $x = 1$, as the price of low-capacity sellers is so high that no buyer visits them. ■

Lemma 5 *The low-capacity sellers' equilibrium price is given by*

$$p_1 = \frac{1 - e^{-\beta} - \beta e^{-\beta}}{1 - e^{-\beta}}$$

Proof. In equilibrium the market utility that a consumer expects is $\bar{U}$. The profit maximisation problem of a low-capacity seller is

$$\max_{p_1} \pi_1 = p_1(1 - e^{-\beta}) \text{ s.t. } (1 - p_1)\frac{1 - e^{-\beta}}{\beta} = \bar{U},$$

or

$$\max_{\beta} \left(1 - \frac{\beta \bar{U}}{1 - e^{-\beta}}\right)(1 - e^{-\beta}),$$

The first-order condition with respect to β yields

$$e^{-\beta} = \bar{U},$$

which we can substitute into p_1 to get

$$p_1 = \frac{1 - e^{-\beta} - \beta e^{-\beta}}{1 - e^{-\beta}}.$$

Note that the second-order condition is $-\beta e^{-\beta} < 0$ which guarantees that p_1 is a good candidate for and equilibrium price, given the buyers' indifference condition. That the price constitutes a unique symmetric equilibrium is shown in many works, eg. Kultti (2011). ■

Lemma 6 *If a high-capacity seller posts price $p > \overline{p}_2$ his expected profit is less than when posting price $\overline{p}_2$.*

Proof. Assume that a high-capacity seller deviates and posts price $p \neq \overline{p}_2$. The queue length γ that this price generates is given by

$$\frac{1 - e^{-\beta}}{\beta}(1 - p_1) = e^{-\beta} = \frac{2 - 2e^{-\alpha} - \alpha e^{-\alpha}}{\alpha}(1 - \overline{p}_2) = \frac{1 - e^{-\gamma}}{\gamma}(1 - p). \quad (17)$$

As $p > \overline{p}_2$ it must be the case that $\gamma < \alpha$. The deviator's profit is less than the equilibrium profit if

$$\left(2 - 2e^{-\gamma} - \gamma e^{-\gamma}\right) p \leq \left(2 - 2e^{-\alpha} - \alpha e^{-\alpha}\right) \overline{p}_2. \quad (18)$$

Substituting from (17) this is equivalent to

$$\left(2 - 2e^{-\gamma} - \gamma e^{-\gamma}\right) \left[1 - \frac{\gamma}{1 - e^{-\gamma}} \frac{2 - 2e^{-\alpha} - \alpha e^{-\alpha}}{\alpha}(1 - \overline{p}_2)\right] \leq \left(2 - 2e^{-\alpha} - \alpha e^{-\alpha}\right) \overline{p}_2.$$

Adopting the notation $\Omega_z \equiv 2 - 2e^{-z} - ze^{-z}$ this, in turn, is equivalent to

$$\Omega_\gamma \left[1 - \frac{\gamma}{1-e^{-\gamma}} \frac{\Omega_\alpha}{\alpha}(1-\overline{p}_2)\right] \leq \Omega_\alpha \overline{p}_2.$$

Next we substitute from (17) for $\frac{\Omega_\alpha}{\alpha}(1-\overline{p}_2)$ to get

$$\Omega_\gamma \left[1 - \frac{\gamma}{1-e^{-\gamma}} e^{-\beta}\right] \leq \Omega_\alpha \overline{p}_2,$$

or

$$\overline{p}_2 \geq \frac{\Omega_\gamma}{\Omega_\alpha}\left[1 - \frac{\gamma}{1-e^{-\gamma}} e^{-\beta}\right].$$

This is equivalent to

$$\frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\alpha}} \geq \frac{\Omega_\gamma}{\Omega_\alpha}\left[1 - \frac{\gamma}{1-e^{-\gamma}} e^{-\beta}\right],$$

which is equivalent to

$$\frac{\Omega_\alpha}{1-e^{-\alpha}}\left(1-e^{-\beta}-\beta e^{-\beta}\right) \geq \frac{\Omega_\gamma}{1-e^{-\gamma}}\left(1-e^{-\gamma}-\gamma e^{-\beta}\right).$$

As $1-e^{-\gamma}-\gamma e^{-\beta}$ reaches its maximum at $\gamma = \beta$ it is enough to show that $\frac{\Omega_\alpha}{1-e^{-\alpha}} > \frac{\Omega_\gamma}{1-e^{-\gamma}}$ but this is the case as $\frac{\Omega_\gamma}{1-e^{-\gamma}}$ is increasing in γ. ■

Lemma 7 *If a high-capacity seller posts price $p < \overline{p}_2$ his expected profit is less than when posting price $\overline{p}_2$.*

Proof. Again (17) holds and we want to show that

$$\left(2-2e^{-\gamma}-\gamma e^{-\gamma}\right)p \leq \left(2-2e^{-\alpha}-\alpha e^{-\alpha}\right)\overline{p}_2. \tag{19}$$

Consider the following process: Find price $q_2^1 < \overline{p}_2$ that generates the same queue length as $\overline{p}_2$, namely $\gamma^0 \equiv \alpha$. Any price $p \in \left[q_2^1, \overline{p}_2\right)$ generates less than equilibrium profits. Then find queue length γ^1 that generates equilibrium profits at price q_2^1. Notice that $\gamma^1 > \alpha$. Next find price q_2^2 that generates queue length γ^1. Clearly $q_2^2 < q_2^1$, and any price $p \in \left[q_2^2, q_2^1\right)$ generates less than equilibrium profits. Find then queue length γ^2 that generates the equilibrium profits at price q_2^2. Notice that $\gamma^2 > \gamma^1$. Find price q_2^3 that generates queue length γ^2. Clearly $q_2^3 < q_2^2$, and any price $p \in \left[q_2^3, q_2^2\right)$ generates less than equilibrium profits. In the sequence $\left\{\left(q_2^i, \gamma^i\right)\right\}$ the first co-ordinate is decreasing and the second increasing and both co-ordinate sequences clearly converge or are not defined: The latter happens if at some point even an infinite queue length (or trading two units with probability one) is not enough to generate equilibrium profits. In this case there does not exist a profitable deviation. So assume that the sequence converges to $(\overline{q}, \overline{\gamma})$.

Consider next price $r_2^1 = 0$. This generates queue length δ^1 such that the buyers are indifferent between contacting any of the sellers. Determine then price r_2^2 that at queue length δ^1 generates the equilibrium profits. Then determine the queue length that price r_2^2 generates as well as price r_2^3 that at queue length δ^2 generates the equilibrium profits. This process gives a sequence $\left\{\left(r_2^i, \delta^i\right)\right\}$ that clearly converges to, say, $\left(\overline{r}, \overline{\delta}\right)$. It is again clear that $\overline{r} \leq \overline{q}$; otherwise there would be prices larger than $\overline{q}$ that would generate profits $\Omega_\alpha \overline{p}_2$, but this is impossible by construction. If $\overline{r} = \overline{q}$ there is no profitable deviation to low prices.

We show that $\overline{r} < \overline{q}$ leads to a contradiction. Notice first that these prices, and the corresponding queue lengths, have to satisfy the following conditions

$$\frac{1 - e^{-\overline{\gamma}}}{\overline{\gamma}}(1 - \overline{q}) = \frac{1 - e^{-\overline{\delta}}}{\overline{\delta}}(1 - \overline{r}) = e^{-\beta},$$

and

$$\Omega_{\overline{\gamma}} \overline{q} = \Omega_{\overline{\delta}} \overline{r} = \Omega_\alpha \overline{p}_2.$$

We can solve

$$\overline{q} = \frac{1 - e^{-\overline{\gamma}} - \overline{\gamma} e^{-\beta}}{1 - e^{-\overline{\gamma}}},$$

$$\overline{r} = \frac{1 - e^{-\overline{\delta}} - \overline{\delta} e^{-\beta}}{1 - e^{-\overline{\delta}}}.$$

Next we show that $\Omega_{\overline{\gamma}} \overline{q} = \Omega_{\overline{\delta}} \overline{r}$ cannot hold. Consider expression $\Omega_z \frac{1-e^{-z}-ze^{-\beta}}{1-e^{-z}}$. Its derivative is

$$e^{-z}(1+z)\frac{1 - e^{-z} - ze^{-\beta}}{1 - e^{-z}} - \Omega_z \frac{e^{-\beta}\left(1 - e^{-z} - ze^{-z}\right)}{\left(1 - e^{-z}\right)^2},$$

which is of the same sign as

$$\left(1 - e^{-z}\right) e^{-z}(1+z)\left(1 - e^{-z} - ze^{-\beta}\right) - \Omega_z e^{-\beta}\left(1 - e^{-z} - ze^{-z}\right).$$

At $z = \beta$ this is negative and continues to be negative for all $z > \beta$. This shows our claim. ∎

The previous results establish that prices $p_1 = \frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\beta}}$ and $\overline{p}_2 = \frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\alpha}}$ constitute an equilibrium.

Lemma 8 *There is a continuum of separating equilibria.*

Proof. The high-capacity sellers are indifferent between posting $p_1 = \frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\beta}}$ and $\underline{p}_2$ if $\underline{p}_2\left(2 - 2e^{-\alpha} - \alpha e^{-\alpha}\right) = p_1\left(2 - 2e^{-\beta} - \beta e^{-\beta}\right)$. Based on the proofs of the previous lemmata it is clear that any price $p_2 \in \left[\underline{p}_2, \overline{p}_2\right]$ constitutes a separating equilibrium with $p_1 = \frac{1-e^{-\beta}-\beta e^{-\beta}}{1-e^{-\beta}}$. Note that the value of β depends on p_2. ∎

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