

# On the Composition Effects on the Labor Market Outcomes <sup>\*</sup>

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## Abstract

It is well documented that hazard rates and entry wages decline with unemployment duration and are procyclical. These outcomes may be explained on the basis of workers' invariant characteristics and composition variation with unemployment duration and over the business cycle. To explore it theoretically and quantitatively, we build a labor market model in which firms lack information on the qualifications of the candidates and react to the asymmetry of information by designing self-selection schemes. As pointed out by the personnel economics literature, performance-contingent compensation plans are central in such sorting schemes. The other key ingredient of our self-selection mechanism is capital-skill complementarity, which allows firms to target one type of applicants by playing with the capital margin. As a result, workers of different types direct their search towards distinct submarkets and high-skilled workers experience higher job-finding rates and entry wages. We calibrate the model to replicate some salient facts of the US economy. In line with the empirical literature, the sorting mechanism explains almost one third of the 48 percent decline of the monthly hazard rate after three months of unemployment relative to the first week. Regarding the procyclical pattern, we show that the composition component of both hazard rates and entry wages barely explains the movements of those variables over the cycle.

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# 1 Introduction

It is well documented that, after controlling for observable characteristics, hazard rates from unemployment and entry wages decline with unemployment duration and are procyclical. These patterns may result from state-contingent mechanisms that deteriorate the job market prospects over duration and during a recession for all job-seekers identically. Alternatively, these outcomes may be explained on the basis of workers' invariant characteristics and composition variation with unemployment duration and over the business cycle. We explore theoretically and quantitatively the latter explanation by building a labor market model in which firms lack information on the qualifications of the candidates and, following Salop and Salop (1976), react to the asymmetry of information by designing self-selection schemes.<sup>1</sup> As pointed out by the personnel economics, e.g. Lazear and Shaw (2007), performance-contingent compensation plans are central in such sorting schemes.<sup>2</sup> The other key ingredient of our self-selection mechanism is capital-skill complementarity, which allows firms to target one type of applicants by playing with the capital margin. As a result, workers of different types direct their search towards distinct submarkets and high-skilled workers experience higher job-finding rates and entry wages. We calibrate the model to replicate some salient facts of the US economy. In line with the empirical literature,<sup>3</sup> the sorting mechanism explains almost one third of the 48 percent decline of the monthly hazard rate after three months of unemployment relative to the first week. Regarding the procyclicality pattern, [...]

To be more precise, we build a directed search model with asymmetric information on workers' skills. We think of such skills as abilities that recruiters cannot grasp from a CV or an interviewing process, but can be assessed on the job. Firms commit to a capital-renting plan. Output is the result of the combination of capital and labor skills, whereas the labor compensation is bargained over. The decentralized assignment of workers to firms is driven by the capital-skill complementarity. That is, a production technology with a positive cross partial derivative leads firms and high-ability

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<sup>1</sup>As they say, firms could alternatively or complementarily design screening devices.

<sup>2</sup>Personnel economics has highlighted that performance pay also aims at selecting workers who can respond to the incentive schemes by exerting more effort. We abstract away from the effort-related agency problem asymmetric information poses, which has been investigated in a frictional labor market by Moen and Rosén (2011).

<sup>3</sup> The empirical literature has emphasized, since Lancaster (1979) and Heckman and Singer (1984), that ex-ante (unobservable) differences account for a large part of the decline in hazard rates over unemployment duration. For example, van den Berg and van Ours (1994, 1996, 1999) conclude that the decline of the hazard rates is mostly due to unobserved heterogeneity.

workers to anticipate that their match would produce a larger surplus and, hence, higher profits and wages. As a result, a relatively larger entry of firms takes place in equilibrium in submarkets targeting high-ability workers leading to higher job-finding rates. Job-seekers who differ in their market ability find it optimal to direct their search to distinct submarkets in equilibrium. This is the case because although low-ability workers would benefit from a higher job-finding rate if applying to submarkets targeting high-skilled workers, the generated surplus would be much lower offsetting the matching gains. The equilibrium is thus block-recursive, which ensures the tractability of the model when adding aggregate shocks to the economy.

To quantitatively analyze the composition effect over the business cycle, we hit the economy with two types of shocks: productivity and separation shocks. We show that the composition component barely accounts for the movements of the job-finding rates and entry wages over the cycle when aggregate productivity shocks are modeled.

We contribute to the literature dealing with asymmetric information in the labor market by exploring the dynamic consequences of performance-based pay schemes as well as exploring one margin (the capital plan) firms may use to upgrade the productivity of their applicants. In a static setting, Michelacci and Suarez (2006) allow firms to either design performance-contingent compensation schemes or commit to a non contingent wage. They find that for a parameter subset both types of wage schemes coexist in equilibrium with high-skilled workers leading their search to performance-pay jobs. One other approach is in Gale (1992) and Guerrieri, Shimer, and Wright (2010) in which workers self-select according to the disutility of work. The former finds that good workers work more hours and are better paid in the absence of frictions. The latter show that this result does not directly extend to a frictional labor market in the sense that good workers do have a higher employability rate, but wages and working time comparisons result from additional assumptions. More specifically, if good workers are not only more productive at any working time, but also increasingly more as hours rise, then they enjoy both higher wages and working time.

In our model the sorting mechanism relies on the complementarity between capital and workers' skills rather than preferences for leisure. Notice that for a sorting mechanism to be consistent with negative duration effects in the labor market it cannot be based on working hours. This is the case because then working time would fall with unemployment duration, a result which is at odds with CPS data.<sup>4</sup> Our modeling may indeed be understood as an extension to unobservable abilities

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<sup>4</sup>Maybe because of institutional reasons, working time is very clustered at 40 hours a week, with almost 47% of

of the complementarity between observable characteristics of the labor input, mainly education, and physical capital that has been widely used in the macroeconomics literature (see e.g. Fallon and Layard (1975) and Krusell, Ohanian, Ros-Rull, and Violante (2000)). On the other hand, by not modeling the working time, we lose the intensive margin the economies use to adjust over the cycles and, hence, limit the analysis to adjustments on the extensive margin.<sup>5</sup> Furthermore, on the technical side, it is worth noting that the Guerrieri, Shimer, and Wright's (2010) set of assumptions and, in particular, their key sorting assumption are not satisfied in our model.

Other articles have also tackled a sorting mechanism. To the best of our knowledge, Lockwood's (1991) random search model is the first in investigating a pure composition effect in hazard rates, although entry wages stay constant in duration. Gonzalez and Shi (2009) and Fernández-Blanco and Preugschat (2012) base their mechanism on symmetric incomplete information on workers' skills and directed search, then state-contingent and composition mechanisms coexist in equilibrium.

The paper is also related to the literature on labor market segmentation. For example, Moen (2003) analyzes a labor market with individual- and match-specific productivity in which firms do not observe the latter whereas firms do observe the overall productivity. He shows that if firms cannot commit to productivity-contingent wages, there are too few skilled jobs posted in equilibrium and, although the wage premium for skilled workers is larger than optimal, their welfare reduces. In our setting, high-ability workers are also worse off in case their low-ability counterparts are willing to enter the submarket identified by the perfect information capital. In such a case, firms commit to larger capital levels to get rid of low-ability applicants leading down the equilibrium wage and job-finding probability.

The rest of the paper is as follows. Section 2 describes the stochastic economy. The equilibrium is characterized in Section 3. Section 4 analyzes the steady state calibrated economy and the implications of the model regarding the average hazard rate and entry wage distributions over unemployment duration. The stochastic results are determined in Section 5. Section 6 concludes and the proofs are postponed to the Appendix.

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the sample -after removing the 10% of individuals who report 'hours vary'-, and at 20 and 30 hours with less than 10% each.

<sup>5</sup>According to Ohanian and Raffo (2012), the volatility of employment is more than twice as large as the one of hours per worker in the US during the 1985-2007 period, whereas employment is slightly more procyclical than hours per worker. van Rens (2012) estimates that the extensive margin accounts for 75% of the adjustment in total hours for the same time period.

## 2 Set Up

In this section, we describe an economy with ex-ante heterogeneity on the supply side of the labor market. There is asymmetric information on the market productivity of workers and idiosyncratic and aggregate shocks.

### 2.1 Description of the Labor Market

The economy is populated by a measure one of workers and a large continuum of firms determined in equilibrium by free entry. All agents are risk neutral and discount future payoffs at common rate  $r$ . Workers differ ex-ante from one another in their market ability. They can be either low-skilled or high-skilled,  $i \in \{\ell, h\}$ . Their market ability is denoted by  $y$ , with  $y_\ell < y_h$ . There is a mass  $\mu_h = \mu$  of high-ability workers.<sup>6</sup> The worker's type is only observable to the worker herself. Time is continuous. The time indices are suppressed for notational simplicity.

**Production and Separation.** Unemployed workers produce  $b$  units of output at home. The market output produced by a job-worker pair at a given instant depends on three objects: the worker's ability  $y$ , the capital level of the firm  $k \in \mathcal{R}_+$ , and the aggregative productivity of the economy  $z$ . The productivity component which is common across all matched pairs belongs to the set  $\mathcal{Z} \equiv \{z_1, \dots, z_n\}$ , with  $z_1 < \dots < z_n$ . Type- $i$  employed workers produce  $f(z, y_i, k)$  in the market. The production technology  $f(\cdot, \cdot, \cdot)$  is increasing and concave in all its arguments with a positive cross partial derivative of capital and skill. Although other functional forms are also valid, to fix ideas we may think of a multiplicative function  $f(z, y, k) = zyk^a$ , with  $a \in (0, 1)$ .

Job-worker pairs see their match destroyed at Poisson rate  $\lambda$ , where  $\lambda \in \Lambda \equiv \{\lambda_1, \dots, \lambda_m\}$ . Let  $\psi \equiv (u_\ell, u_h, z, \lambda)$  define the aggregate state of the economy, where  $u_i$  stands for the unemployment rate of type- $i$  workers. The stochastic process of  $\psi$  is Markovian,  $\pi(\psi'|\psi)$ . The law of motion of the unemployment rates will be determined later. Let  $\Psi \equiv [0, 1]^2 \times \mathcal{Z} \times \Lambda$  denote the set of possible values of the aggregate state of the economy.

**Capital Plans and Commitment. Search and Matching.** Firms and workers direct their search to a given submarket. Search is directed in the sense that agents anticipate that a larger entry of workers and hence a higher job-filling rate takes place in submarkets promising higher

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<sup>6</sup>Hence, the mass of type- $\ell$  workers is  $\mu_\ell = 1 - \mu$ . Their market productivity is  $y_\ell$ .

employment prospects. Commitment is short-term in the sense that firms commit to a capital level  $k$  given the current state of the economy  $\psi$  instead to an aggregate state-contingent capital plan. Moreover, as in Michelacci and Suarez (2006), we assume that neither type- nor output-contingent contracts are enforceable.<sup>7</sup> That is, a submarket is defined by one capital level. Upon the arrival of an aggregate shock, the firm-worker pair renegotiate and agree to choose the capital level that maximizes the returns. Then, workers locate themselves in submarkets that maximize their expected discounted utility. Firms incur the capital rental price  $r$  plus the depreciation cost at rate  $\delta$  at any instant while producing. The returns from capital investment are shared by the firm and the worker as wages are bargained over. Let  $\alpha$  denote the worker's bargaining power. However, because of the capital renting and the depreciation costs, larger capital levels do not monotonically translate into higher wages. Instead, the flow returns  $p(z, y, k) \equiv f(z, y, k) - (r + \delta)k$  show an inverted U shape as a function of  $k$ . Let  $\bar{k}_i(\psi)$  denote the capital level that maximize the flow returns of type- $i$  workers, respectively. That is,  $p_k(z, y_i, \bar{k}_i(\psi)) = 0$ . To guarantee existence of equilibrium at all possible states, we assume that  $\lim_{k \rightarrow 0} f_k(z, y, k) = \infty$ ,  $\lim_{k \rightarrow \infty} f_k(z, y, k) = 0$ , and  $b < \max_k p(z_1, y_\ell, k)$ .

Let  $\Theta(k, \psi)$  denote the market tightness defined as the ratio of vacancies to workers in submarket  $k$  when the state variable is  $\psi$ . This ratio must be consistent with the agents' optimizing behavior in equilibrium, which will be further explained below. A firm fills a vacancy in submarket  $k$  at Poisson rate  $\eta(\Theta(k, \psi))$ . Workers of either type find jobs at equal rate  $\nu(\Theta(k, \psi))$  because of the asymmetric information assumption about the applicant's skills. Functions  $\eta$  and  $\nu$  are decreasing and increasing in their capital argument, respectively. Again, to guarantee existence of equilibrium, we assume that  $\lim_{\theta \rightarrow 0} \eta(\theta) = \lim_{\theta \rightarrow \infty} \nu(\theta) = \infty$  and  $\lim_{\theta \rightarrow \infty} \eta(\theta) = \lim_{\theta \rightarrow 0} \nu(\theta) = 0$ . Because the mass of newly employed workers must coincide with the mass of newly filled jobs, it can be shown that the job-finding rate becomes  $\nu(\Theta(k, \psi)) = \Theta(k, \psi)\eta(\Theta(k, \psi))$ .

**Timing.** The timing of the events is as follows. At the beginning of every instant, firms hold one single job and can be either vacant or actively producing, whereas workers are either employed or unemployed. There is potentially a continuum of submarkets indexed by a capital level  $k$ . Without loss of generality, we assume that a submarket is identified by an state-contingent capital level  $k$ . A submarket where there are firms and workers seeking a trading partner is said to be active. Vacant

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<sup>7</sup>By non-enforceability, we mean that worker's type and output are not verifiable by a third party. Subsection 3.4 discusses the differences with respect to equilibria with state-contingent capital plans and with revelation mechanisms.

firms enter a submarket where to locate their vacancy at cost  $c$ . Then, unemployed workers choose a submarket to search for a job. Matching and production takes place. Wages are continuously bargained over upon matching. Job-worker pairs see their match destroyed at Poisson rate  $\lambda$ . When this occurs, the firm vanishes and the worker becomes newly unemployed. At the end of the period, a shock hits the economy at Poisson rate  $\varphi$  and the new aggregate productivity or separation rate is drawn by Nature. New capital levels are also renegotiated at ongoing productive pairs.

**Value Functions.** An unemployed worker of type  $i$  derives utility from home production  $b$ . She chooses the submarket that maximizes her utility and becomes employed at rate  $\nu(\Theta(k, \psi))$  obtaining value  $\alpha S_i(k, \psi)$ . Her continuation value switches to the expected value  $\mathbb{E}_\psi(U_i(\psi'))$  if an aggregate shock arrives, an event that occurs at rate  $\varphi$ . The expectations are always conditional on the current aggregate productivity.

$$rU_i(\psi) = b + \max_k \nu(\Theta(k, \psi))\alpha S_i(k, \psi) + \varphi(\mathbb{E}_\psi(U_i(\psi')) - U_i(\psi)) \quad (1)$$

The (net) surplus, or value of a job-worker pair net of their outside options,  $S_i(k, \psi)$ , consists of the output net of capital-renting and -depreciation costs as well as unemployment value plus the continuation value of the match. It satisfies the following functional equation<sup>8</sup>

$$(r + \lambda + \varphi)S_i(k, \psi) = p(z, y_i, k(\psi)) - (r + \varphi)U_i(\psi) + \varphi\mathbb{E}_\psi(U_i(\psi') + S_i(\bar{k}_i(\psi'), \psi')) \quad (2)$$

Firms choose a submarket to post their vacancies to maximize expected profits. A vacant firm incurs one time cost  $c$  when posting the vacancy in submarket  $k$ . The job can be filled by either type- $h$  or - $\ell$  workers. Let  $\rho_i(k, \psi)$  denote the proportion of workers of type- $i$  in submarket  $k$  given state  $\psi$ . By definition, it must be the case that the vector  $(\rho_h(k, \psi), \rho_\ell(k, \psi))$  belongs to the simplex  $\Delta^1$ . The returns of a filled vacancy are  $(1 - \alpha)S_i(k, \psi)$ . Thus, the value of the vacant firm is defined by

$$V(\psi) = -c + \eta(\Theta(k, \psi)) \sum_i \rho_i(k, \psi)(1 - \alpha)S_i(k, \psi) \quad (3)$$

Furthermore, expected profits must be zero in any active submarket in equilibrium because of free entry,  $V = 0$ . Therefore, a capital plan  $k$  that maximizes the surplus of their relationship is mutually beneficial for workers and firms. Nevertheless, firms may face workers of different types

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<sup>8</sup>The derivation of the surplus expression is left to the Appendix Subsection 8.1. As an abuse of language, we often use the term surplus to refer to the net surplus.

in a given submarket  $x$  and then will have to average the given surpluses with their respective matching probabilities.

Given the state  $\psi$ , the law of motion of the unemployment rates of low- and high-ability workers is

$$\dot{u}_i = \lambda(1 - u_i) - \nu(\Theta(k_i(\psi), \psi))u_i, \text{ for } i \in \{\ell, h\} \quad (4)$$

### 3 Equilibrium

We turn to define the equilibrium allocation. Although a natural extension of the equilibrium concept defined in the competitive search literature, we benefited from Gale 1992 and Guerrieri et al 2010.

**Definition 1** *A stationary equilibrium consists of a state-contingent distribution  $G_\psi$  of vacancies in active submarkets with support  $\mathcal{X}_\psi \subset X$  and  $\psi \in \Psi$ , a pair of value functions  $U_i : \Psi \rightarrow \mathcal{R}_+$ , a pair of surplus values  $S_i : X \times \Psi \rightarrow \mathcal{R}_+$ , a tightness function  $\Theta : X \times \Psi \rightarrow \mathcal{R}_+$ , a function  $\rho : X \times \Psi \rightarrow \Delta^1$  such that*

1. *Firms' profit-maximizing program and free entry:*

$$\forall k \in X \text{ and } \psi \in \Psi, \eta(\Theta(k, \psi)) \sum_i \rho_i(k, \psi)(1 - \alpha)S_i(k, \psi) \leq c, \text{ with equality if } k \in \mathcal{X}_\psi.$$

2. *Workers' optimal search:  $\forall \psi \in \Psi$ ,*

*the type- $i$  worker's value satisfies*

$$rU_i(\psi) = b + \max_{k \in \mathcal{X}_\psi} \nu(\Theta(k, \psi))\alpha S_i(k, \psi) + \varphi(\mathbb{E}_\psi(U_i(\psi')) - U_i(\psi))$$

*Furthermore,  $\forall k \in X_\psi$ ,  $rU_i(\psi) \geq b + \nu(\Theta(k, \psi))\alpha S_i(k, \psi) + \varphi(\mathbb{E}_\psi(U_i(\psi')) - U_i(\psi))$ , with equality if  $\Theta(k, \psi) < \infty$ , and  $\rho_i(k, \psi) > 0$*

*If  $S_i(k, \psi) < 0$ , then either  $\Theta(k, \psi) = \infty$  or  $\rho_i(k, \psi) = 0$ .*

3. *The surplus values  $S_i$  satisfy the functional equation (2).*

4. *Market-clearing condition:  $\forall \psi \in \Psi$ ,*

$$\int_{\mathcal{X}_\psi} \frac{\rho_i(k, \psi)}{\Theta(k, \psi)} dG_\psi(k) = u_i(\psi)\mu_i, \forall i$$

The interpretation of the equilibrium definition regarding active submarkets is standard. The first equilibrium condition guarantees that, first, vacant firms maximize profits and, second, the null expected profits of a vacant job because of free entry. Whereas the second condition establishes the optimal search behavior of workers given the distribution of active markets. The third condition determines equilibrium wages and, finally, the sum of job-seekers across active submarkets must be equal to the mass of unemployed workers for each type.

Let us turn to describe the off-the-equilibrium beliefs that support the equilibrium allocation by considering a trembling hand kind of argument. Think of an arbitrarily small mass of firms that consider to deviate and locate their vacancy in a given submarket  $k'$ . Those firms form rational expectations about the search behavior of all workers in the continuation *subgame* determining the triple  $(\Theta(k', \psi), \rho_\ell(k', \psi), \rho_h(k', \psi))$ . The second equilibrium condition establishes that the expectations on  $\Theta(k', \psi)$  are pinned down by the workers who are willing to accept the lower tightness, determining in turn the  $\rho$  proportions, provided that the employment value promised by  $k'$  is large enough. In other words, if a mass of firms deviated to  $k'$ , then there would be a flow of workers from the equilibrium submarkets until their utility falls down to their market value. This subgame perfection condition explains why the system of rational beliefs is determined by the type who is indifferent between applying to submarket  $k'$  and the equilibrium one.

### 3.1 Characterization of Equilibrium.

The following proposition claims that there cannot exist an equilibrium in which both types of workers apply to the same job offers. The result is very intuitive. If a submarket  $k$  were active with  $\rho_\ell(k, \psi), \rho_h(k, \psi) > 0$ , there would be a profitable deviation of firms committing to a capital level  $k'$  arbitrarily smaller or bigger than  $k$  for the current aggregate state. By entering submarket  $k'$ , firms would manage to get rid of the applicants with lower surplus leading to a discrete jumps in profits.

**Proposition 3.1** *There is no equilibrium in which workers of different types search in the same active submarket. That is, for all active submarket  $k \in \mathcal{X}_\psi$  in equilibrium, it is the case that either  $\rho_\ell(k, \psi) = 1$  and  $\rho_h(k, \psi) = 0$  or vice versa.*

As a result, conditional on existence, the decisions made by the agents in equilibrium depend on the state of the economy only through the aggregate productivity and the separation rate, but

not on the unemployment rate of each type of worker. This feature has been referred to in the literature as block-recursivity and it simplifies much the characterization of the equilibrium as the equilibrium objects are jump variables that vary over a finite set,  $\mathcal{Z}$ .

The following lemma states that in any submarket where high-ability workers are willing to enter the surplus derived from them is larger than the one linked to their low-ability counterparts. It follows that the value of the firm from hiring a type- $\ell$  worker at these submarkets is lower than filling the vacancy with a type- $h$  worker as  $(1-\alpha)S_\ell(k, \psi) < (1-\alpha)S_h(k, \psi)$ . As a result, this lemma partially resembles the monotonicity assumption in Guerrieri, Shimer, and Wright (2010), according to which principals (firms) always prefer to trade with higher types (higher-ability workers).<sup>9</sup>

**Lemma 3.2** *Let  $k$  be a submarket such that  $\rho_\ell(k, \psi) \geq 0$ ,  $\rho_h(k, \psi) > 0$  and  $\Theta(k, \psi) < \infty$ . Then,  $S_\ell(k, \psi) < S_h(k, \psi)$ .*

To characterize the equilibrium we proceed in several steps. We first focus on the necessary conditions the equilibrium allocation must satisfy regarding type- $\ell$  workers and then with regard to high-ability workers. Second, we establish that such conditions are also sufficient and state the existence of equilibrium. The following result claims that all low-ability workers enter the same submarket, if any, which is the surplus-maximizing capital level,  $\bar{k}_\ell(\psi)$ .

**Proposition 3.3** *Let  $\{G_\psi, \mathcal{X}_\psi, \Theta(\cdot, \psi), \{U_i(\psi), S_i(\cdot, \psi), \rho_i(\cdot, \psi)\}_i\}_\psi$  be an equilibrium. Let  $\psi$  be the state of the economy. If there exists an active submarket  $k_\ell(\psi) \in \mathcal{X}_\psi$  such that  $\theta_\ell(\psi) \equiv \Theta(k_\ell(\psi), \psi) < \infty$  and  $\rho_\ell(k_\ell(\psi), \psi) > 0$ , then  $\rho_\ell(k', \psi) = 0$  for all  $k' \in \mathcal{X}_\psi \setminus \{k_\ell(\psi)\}$ . Let  $S_\ell(\psi) \equiv S_\ell(k_\ell(\psi), \psi)$  denote the surplus derived at submarket  $k_\ell(\psi)$ . The vector  $(\theta_\ell(\psi), k_\ell(\psi), S_\ell(\psi))_\psi$  is a*

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<sup>9</sup>The assumption of capital readjustment at the arrival of an aggregate shock is primarily made to permit us to compare the returns from the two types of workers at any given submarket where skilled workers are willing to enter. Guerrieri, Shimer, and Wright (2010) make the monotonicity assumption to somehow deal with this problem. Although this assumption is unnecessary for the result in Proposition 3.1 or even to characterize the economy with no aggregate shocks, we do need such a comparison to characterize the equilibrium in the business cycle setup. Although it may seem somewhat natural that ongoing pairs readjust their capital decisions with the arrival of an aggregate shock, there is a caveat: the undesirable assignment to the rate  $\varphi$  of the double role of aggregate productivity and capital-readjustment. We see it as a compromise and will argue how to estimate  $\varphi$  in the quantitative exercise.

solution of the following system of equations (5)-(7)

$$c = \eta(\theta(\psi))(1 - \alpha)S_\ell(\psi), \quad \forall \psi \in \Psi \quad (5)$$

$$\frac{r+\lambda+\varphi}{\eta(\theta(\psi))} = (p(z, y_\ell, k(\psi)) - b) \frac{1-\alpha}{c} - \alpha\theta(\psi) + \varphi\mathbb{E}_\psi \left( \frac{1}{\eta(\theta(\psi'))} \right), \quad \forall \psi \in \Psi \quad (6)$$

$$k(\psi) = \bar{k}_\ell(\psi), \quad \forall \psi \in \Psi \quad (7)$$

This set of necessary conditions (5)-(7) stems from the well-known equivalence to the problem of maximizing the utility of low-ability workers subject to guaranteeing non-negative profits to firms (see e.g. Acemoglu and Shimer (1999a)). That is, given the aggregate state of the economy  $\psi$ , the equilibrium objects of the low-ability market are determined by the following program:

$$\begin{aligned} \max_{\theta(\psi), S_\ell(\psi)} \quad & \nu(\theta(\psi))S_\ell(\psi) \\ \text{s. to} \quad & \eta(\theta(\psi))(1 - \alpha)S_\ell(\psi) \geq c \end{aligned} \quad (8)$$

The equations (5) constitute the zero-profit conditions, whereas the set (7) stands for the first-order condition with respect to capital,  $p_k(z, y_\ell, k) = 0$ . They deliver the values of the surplus linked to low-ability workers, given the tightness, and the capital level, respectively. The equilibrium conditions (6) in turn come out from replacing the surplus values from the functional equations (2) by using the zero-profit condition (5).

We now turn to high-ability workers. Let  $\psi \in \Psi$  be a state of the economy. Given the vector  $(U_i(\psi'))_{\psi'}$ , we define problem  $(P(\psi))$  as

$$\begin{aligned} \max_{\theta, k} \quad & \nu(\theta)\alpha S_h(k, \psi) \\ \text{s. to} \quad & \eta(\theta)(1 - \alpha)S_h(k, \psi) \geq c \\ & b + \nu(\theta)\alpha S_\ell(k, \psi) + \varphi\mathbb{E}_\psi(U_\ell(\psi)) \leq (r + \varphi)U_\ell(\psi) \end{aligned}$$

where the functions  $S_i(\cdot, \psi)$  are defined by (2). Notice that the problem for high-ability workers has one extra restriction relative to problem (8). This is the case to guarantee that low-ability workers do not enter the type- $h$  submarket. To help us with the characterization of equilibrium, we define a solution of a bigger problem.

**Definition 2** We say that a vector  $(\theta(\psi), k(\psi), U_h(\psi))_\psi$  is a solution of problem  $(P)$  if

1. Given  $(U_\ell(\psi))_\psi$  and  $(U_h(\psi))_\psi$ ,  $(\theta(\psi), k(\psi))$  is a solution of problem  $(P(\psi))$ ,  $\forall \psi \in \Psi$ .

2. The following equations hold:

$$b + \nu(\theta(\psi))\alpha S_h(k(\psi), \psi) + \varphi \mathbb{E}_\psi (U_h(\psi')) = (r + \varphi)U_h(\psi), \quad \forall \psi \in \Psi$$

Now, we state that an equilibrium must be a solution of problem (P).

**Proposition 3.4** *Let  $\{G_\psi, \mathcal{X}_\psi, \Theta(\cdot, \psi), \{U_i(\psi), S_i(\cdot, \psi), \rho_i(\cdot, \psi)\}_i\}_\psi$  be an equilibrium. Consider a vector  $(k(\psi))_\psi$  of active submarkets such that  $\theta(\psi) \equiv \Theta(k(\psi), \psi) < \infty$  and  $\rho_h(k(\psi), \psi) > 0$ , then the equilibrium vector  $(\theta(\psi), k(\psi), U_h(\psi))_\psi$  is a solution of problem (P), given  $(U_\ell(\psi))_\psi$ .*

The following result states necessary conditions for the equilibrium allocation to satisfy.

**Proposition 3.5** *There exists a solution  $(\theta(\psi), k(\psi), U_h(\psi))_\psi$  of problem (P). Such a solution satisfies the following conditions for all  $\psi \in \Psi$ :*

$$c = \eta(\theta(\psi))(1 - \alpha)S_h(k(\psi), \psi) \tag{9}$$

$$\frac{r + \lambda + \varphi}{\eta(\theta(\psi))} = (p(z, y_h, k(\psi)) - b)\frac{1 - \alpha}{c} - \alpha\theta(\psi) + \varphi\frac{1 - \alpha}{c}\mathbb{E}_\psi (S_h(\bar{k}_h(\psi'), \psi')) \tag{10}$$

$$b + \nu(\theta(\psi))\alpha S_h(k(\psi), \psi) + \varphi \mathbb{E}_\psi (U_h(\psi')) = (r + \varphi)U_h(\psi), \tag{11}$$

where

$$k(\psi) = \bar{k}_h(\psi), \text{ if } b + \nu(\theta(\psi))\alpha S_\ell(k(\psi), \psi) + \varphi \mathbb{E}_\psi (U_\ell(\psi')) \leq (r + \varphi)U_\ell(\psi) \tag{12}$$

Otherwise,

$$k(\psi) > \bar{k}_h(\psi) \text{ and } b + \nu(\theta(\psi))\alpha S_\ell(k(\psi), \psi) + \varphi \mathbb{E}_\psi (U_\ell(\psi')) = (r + \varphi)U_\ell(\psi). \tag{13}$$

Figure 3.1 shows the two possible equilibrium outcomes resulting from conditions (12) and (13). The left panel shows the case in which low-ability workers are not willing to enter the submarket  $\bar{k}_h$ . In this case, the indifference curve of each type of workers is tangent to the isoprofit curve of the firms. That is, the first case ends up being the perfect information scenario. Instead, the right subfigure depicts the scenario in which type- $\ell$  workers are better off at such submarket. As a result, firms are willing to incur an excess of capital level forgoing potential returns from matching type- $h$  workers to prevent low-ability workers from applying. As Guerrieri, Shimer, and Wright (2010) put it, firms commit to the less expensive contract that guarantees a selected pool of applicants. The forgone gains represented by the capital difference  $k_h - \bar{k}_h$  and its associated smaller entry

of firms targeting high-ability workers amount to the social cost of asymmetric information. The forgone gains can also be interpreted as the burden the presence of low-ability workers creates on their high-ability counterparts. Because of the assumption that contracting terms are renegotiated whenever an aggregate shock hits the economy, the forgone gains are not spread over the lifecycle of the job, but limited to the current state of the economy.

Whether the equilibrium outcome is the first best allocation depends crucially on the arrival rate of the aggregate shock  $\varphi$  and the capital elasticity of the production function. Regarding the former, the larger  $\varphi$ , the larger the expected surplus in the type- $h$  submarket for skilled workers; hence, the larger entry of firms. This together with an also larger surplus for unskilled workers makes the latter more willing to enter this submarket and moves the equilibrium away from the first best. Regarding the effect of the parameter  $a$ , recall that capital commitment is the margin firms have to screen out unskilled workers. For low values of capital elasticity, firms need to commit to very high capitals levels to screen out unskilled applicants. In the extreme case, when production is perfectly inelastic in capital,  $a = 0$ , firms would have to commit to  $k = \infty$ , i.e. they have no means to separate candidates. To put it differently, if the skill-related difference of the marginal productivity of capital, i.e. the cross-partial derivative of the production function, is increasing in the elasticity  $a$ , then it becomes less costly for firms to screen out unskilled applicants the higher the parameter  $a$ .

Now, we show two final results to claim existence of the equilibrium allocation.

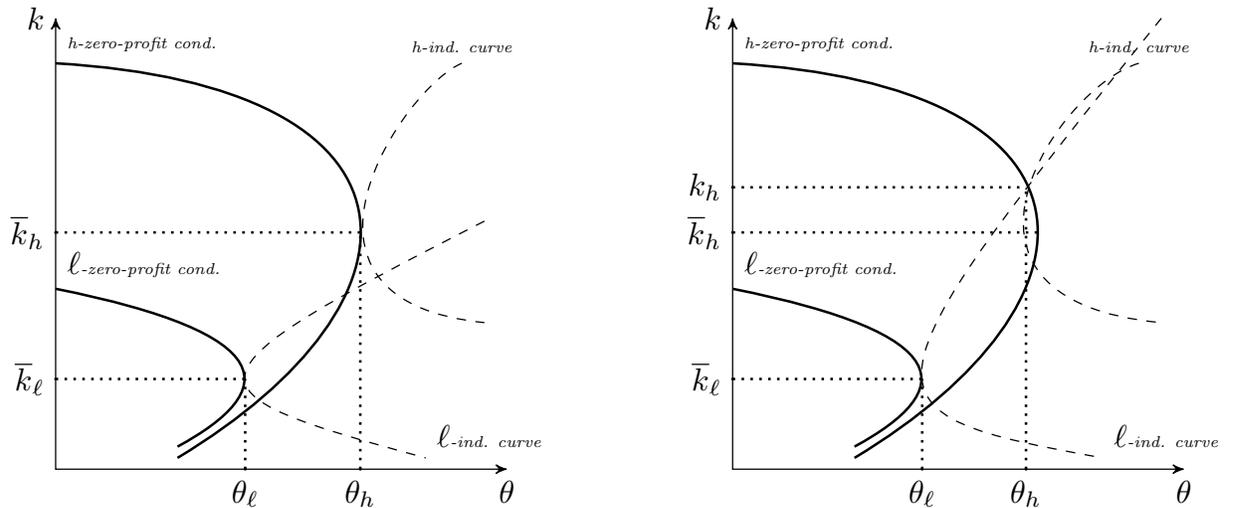


Figure 1: The two possible separating equilibrium allocations.

**Proposition 3.6** *Let  $(\theta_\ell(\psi), k_\ell(\psi), S_\ell(\psi))_\psi$  be the solution of the system (5)-(7), and  $(U_\ell(\psi))_\psi$  be defined as the solution of the functional equation (1) evaluated at the given vector. Given  $(U_\ell(\psi))_\psi$ , let  $(\theta_h(\psi), k_h(\psi), U_h(\psi))_\psi$  be a solution of problem (P), and  $(S_h(\psi))_\psi$  denote the associated surplus. Then, there exists an equilibrium such that, given state  $\psi \in \Psi$ , the set of active submarkets  $\mathcal{X}_\psi \equiv \{k_\ell(\psi), k_h(\psi)\}$ , the unemployment value is  $U_i(\psi)$  and the surplus  $S_i(k_i(\psi), \psi) = S_i(\psi)$ , the tightness of the active submarkets  $\Theta(k_i(\psi), \psi) = \theta_i(\psi)$ , and the beliefs  $\rho_i(k_i(\psi), \psi) = 1$  for  $i \in \{\ell, h\}$ .*

**Corollary 3.7** *There exists an equilibrium.*

### 3.2 Equilibrium Wages and Comparison across skills.

Upon matching, a firm and a worker set the wage for the period, which will be renegotiated inasmuch as a shock hits the economy. Wages are Nash-bargained. Let  $\alpha$  denote the bargaining power of the worker. Workers obtain a proportion  $\alpha$  of the surplus of the match in addition to the unemployment value  $rU_i(\psi)$ .

That is, the equilibrium wages are determined as<sup>10</sup>

$$w_i(k_i(\psi), \psi) = p(z, y_i, k_i(\psi)) - (1 - \alpha) \left( (r + \lambda + \varphi) S_i(k_i(\psi), \psi) - \varphi \mathbb{E}_\psi (S_i(\bar{k}_i(\psi'), \psi')) \right) \quad (14)$$

The following proposition states that high-ability workers enjoy larger employment prospects along all dimensions relative to their low-ability counterparts.

**Proposition 3.8** *The equilibrium capital levels, tightness, job-finding rates, surpluses, unemployment values and entry wages are larger for high-ability workers.*

Moreover, this result provides one simple intuition to understand why high-ability workers have no incentives in applying to low-type submarkets and, hence, the difference in the constraint set of problem  $(P(\psi))$  and its low-ability counterpart. The returns from searching in the type- $\ell$  submarket,  $\nu(\theta_\ell(\psi))\alpha S_h(x_\ell, \psi)$ , are strictly lower than the returns of searching in the high-ability submarket,  $\nu(\theta(\psi))\alpha S_h(x_\psi, \psi)$ .

### 3.3 Constrained Efficiency

When considering constrained efficiency, there are two margins to look at: the capital levels and the entry of firms. Because of the wage-bargaining setting, standard wage-related inefficiencies

<sup>10</sup>The equilibrium wages are derived in detail in the Appendix Subsection 8.1.

may arise in equilibrium regarding the entry of firms. More specifically, it is easy to show that if the Hosios condition holds, this second margin is efficiently set at the unskilled submarket.<sup>11</sup> However, a capital-related inefficiency may result in equilibrium because of the asymmetric information assumption. Recall that the left panel of Figure 3.1 depicts the equilibrium allocation in which there is no economic cost from the asymmetric information assumption, whereas the right one stands for the case in which firms and high-skilled workers choose a higher capital level than the first best to screen out low-skilled candidates. Considering this latter equilibrium, it might seem that a straightforward Pareto improvement could be obtained by means of a system of transfers from high- to low-skilled unemployed in order for the latter to restrain themselves from applying to the high-skilled submarket  $\bar{k}_h$ . However, this is not the case because of informational asymmetry and, hence, there is no way to sustain this alternative allocation as low-skilled agents cannot be identified. Moreover, the relative size of low-skilled unemployed workers should be small enough for the output gains to outweigh the total amount of such transfers.

One other way to look at the relevance of the relative size argument follows Guerrieri, Shimer, and Wright (2010). As they show, the equilibrium depicted by the right panel of Figure 3.1 is Pareto-dominated by a pooling allocation at submarket  $\bar{k}_h$  if the relative size of the low-skilled unemployed workers is sufficiently small. The key aspect of the inefficiency result is that the cost of the screening out of low-skilled applicants does not depend on the relative size of this group. Then, as the social returns of posting vacancies in the pooling allocation are continuous in the size of the low-skilled unemployment, such social returns become larger in the pooling allocation if the share of low-skilled unemployed workers is sufficiently small.

Considering the size issue, we can also argue that higher unemployment benefits may yield efficiency gains. An increase in the parameter  $b$  has a number of implications. First, such an increase reduces the surpluses for all matches, *ceteris paribus*. Second, the second constraint of problem ( $P(\psi)$ ) becomes less binding. This implies that, unlike the low-skilled case, the surplus linked to a high-skilled worker may increase due to firms committing to a lower capital level. Provided that the surplus generated by high-skilled workers increased, there would be a larger entry of firms. The potential overall effect on the high-skilled submarket may overcome the negative effects on low-skilled workers, particularly for a sufficiently small mass of low-skilled workers.

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<sup>11</sup>See e.g. Acemoglu and Shimer (1999b).

### 3.4 Mechanisms and Contracts

For expositional reasons, we have assumed that type- or outcome-contingent capital contracts are not enforceable. Likewise, contracts do not specify different capital levels for different aggregate states because of tractability reasons. That is, firms can only commit to a single capital level. The contracting space analyzed in the previous sections appears thus far from being complete. We now discuss the implications of these practical restrictions.

Given the current state  $\psi$ , consider the setup in which firms can commit to mechanisms such as  $(k_\ell(\psi), k_h(\psi))$ . By the Revelation Principle, we can focus on incentive compatible mechanisms. It is straightforward to extend to our environment the result in Guerrieri, Shimer, and Wright (2010) regarding the equivalence of equilibria with contracts and with revelation mechanisms. As the authors say, it is central in this equivalence result the fact that firms have the means to separate applicants in different submarkets in both environments.

However, limiting the analysis to the contracting environment of the previous sections may seem restrictive inasmuch as contracts stipulating capital levels contingent on aggregate economic conditions are not considered. We conjecture that for any equilibrium with aggregate state-contingent contracts, there must be an equilibrium allocation in our setup which is payoff-equivalent. Certainly, there is no difference in both environments for low-ability workers. Our understanding is that the key aspect is once again that firms have the instruments of separating applicants in different submarkets. Such screening-out of unskilled candidates may yield some costs as the submarket for high-ability workers may be shifted away from the first best allocation. Such costs may be concentrated along the current state or spread over the lifetime of the relationship. Because of not having on the job search along with risk neutrality on both sides of the market, we think that the timing of such relative losses plays no theoretical role, in particular on the entry of firms. Nevertheless, it does have implications for our quantitative analysis of wages in Section 6 as capital levels map into wages and we are not quantitatively assessing lifetime earnings but wage cyclicity.

## 4 Calibration

In this section, we briefly comment on the actual data we use in our quantitative exercise and, then, describe the calibration details.

## 4.1 CPS Data

We derive the actual duration distributions of US monthly hazard rates and entry wages plotted in Figure 5.2 as well as the time series of these two variables and some of the calibration targets from the Current Population Survey (CPS) for the 1994-2011 period. Broadly speaking, we restrict the sample to individuals of the CPS outgoing rotation groups, aged 20 to 60 who were active in the labor market in the previous interview and stay active in the reporting month. We exclude workers on temporary layoff and those with farming-, army- and public sector-related jobs. The reason to exclude temporary layoffs is because their search behavior and prospects differ much from permanently laid-off workers. See e.g. Pries and Rogerson (2005) for a similar approach. Using SIPP data, Fujita and Moscarini (2012) document that the recall rates for temporarily laid-off unemployed stay above 85% since 1996. Katz and Meyer (1990) show that the search intensity and job-finding rates and wages largely differ for workers with recall expectations relatively to those with no expectations. Unemployed workers on temporary layoff account for 14.65% of total unemployment in our sample. The targets we use in the calibration will be adjusted properly.

To remove the (composition) effects stemming from observable characteristics, we deal with a number of controls such as experience, unemployment rate of the month and a number of dummies defined for example by industry, occupation, race, marital status, and gender. Following the labor literature regarding the job-separation and job-finding probabilities at the steady state, we run a probit regression, whereas we run a Mincerian regression for log wages.<sup>12</sup> The duration distributions depicted in Figure 5.2 show the predicted hazard rates and entry wages at each duration computed by replacing the unemployment duration variable by the corresponding duration value and the remaining variables by their means.

To construct time series of entry wages, hazard rates and separation rates, we cannot follow the same strategy as the composition of the pool of individuals may change over time and evaluating the remaining variables at the means would not exclude the composition effects due to observables. Then, we run linear regressions without a year variable and controlling for neither unemployment rate nor unemployment duration as the latter could mask some unobservable characteristics. We compute the monthly averages of the residuals and obtain the quarterly averaged series. Finally, we HP filter it with the usual smoothing parameter 1600. Table 2 reports the correlation with productivity and volatility of the variables.

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<sup>12</sup>See e.g. ?.

To hit the model with aggregate productivity shocks, we use data on output per job of the non-farm business sector taken from the Bureau of Labor Statistics. We use quarterly, seasonally adjusted series from 1947 to 2012 and apply the HP filter with parameter 1600. For consistency reasons, we then restrict our sample to 1994-2011 to compute the statistics. We run a regression of the productivity on its lagged value to compute the autocorrelation and the standard deviation of the residuals.

## 4.2 Calibration

As is common in the literature, our calibration procedure is in two steps. First, we target a number of acyclical patterns and, second, we hit the calibrated economy with the aggregate shocks and calibrate the associated parameters to replicate the aggregate productivity distribution observed in the data. A period is set to be a month. We assume the production function to be multiplicative in the worker’s ability and capital,  $f(z, y, k) = zyka^a$ , with  $a < 1$ . We normalize the market productivity of the skilled to  $y_h = 100$ .<sup>13</sup> Following the literature, the matching technology is Cobb-Douglas, i.e.  $\eta(\theta) = \Gamma\theta^{-\gamma}$  and  $\nu(\theta) = \Gamma\theta^{1-\gamma}$ .

Table 1 lists the thirteen parameters to be calibrated. The discount factor  $r$  is set to match a 4% annual interest rate. For the elasticity of the matching technology, we use the standard value  $\gamma = 0.5$ , which lies within the range surveyed in Petrongolo and Pissarides (2001). Backus, Henriksen, and Storesletten (2008) report that the depreciation rate has steadily increased, rising from 3% in 1970 to 4.15% in 2004 for government capital and from 5.25% to 8.50% for business capital, whereas the rate for private residential capital has remained constant and equal to 1.5%. We take the intermediate annual value of 6%, which implies a monthly rate  $\delta = 0.0051$ .

Regarding the separation rate, the average monthly EU transition rate is 0.0086 in our dataset after excluding workers on temporary layoff. This number is much lower than those reported in the literature. For example, Shimer (2005b) estimates a monthly separation rate at 0.035 using CPS data for the 1951-2003 period. Our numbers are instead very close to those reported by Fujita and Moscarini (2012) for the same time period conditioning on permanent separations and also by Pries and Rogerson (2005). For consistency reasons, we prefer the average of the probit predicted probabilities after controlling for observables characteristics, which implies a Poisson rate

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<sup>13</sup>Indeed, the relevant object is the ratio  $y_h/y_\ell$ . If both objects are factored by  $z$ , then multiplying  $b$  and  $c$  by  $z^{1/(1-a)}$  delivers the same steady state equilibrium tightness  $\theta_h$  and  $\theta_\ell$  and equal relative wages.

Table 1: Calibration

Parameter	Description	Value	Source/Target
Exogenously Set Parameters			
$y_h$	Productivity of skilled	100	normalization
Individually Calibrated Parameters			
$r$	Discount factor	0.0033	4% annual interest rate
$\delta$	Depreciation rate	0.0051	Backus, Henriksen, and Storesletten (2008)
$\gamma$	Elasticity job-filling rate	0.5	Petrongolo and Pissarides (2001)
$\lambda$	Separation rate	0.0090	CPS data
Jointly Calibrated Parameters			
$\alpha$	Workers' bargaining power	0.5984	tightness = 0.4142 <sup>(1,2)</sup>
$y_l$	Productivity of unskilled	57.1071	vacancy duration = 1.25
$\mu$	Share of high-skilled	0.2198	st dev of standardized log wages = 0.3285 <sup>(2)</sup>
$\Gamma$	Scale job-filling rate	0.6552	average job-finding rate = 0.28 <sup>(2)</sup>
$a$	Production elasticity	0.2626	capital-output ratio = 12·2.6 <sup>(2)</sup>
$c$	Vacancy cost	283.0984	13% quarterly average wage <sup>(3)</sup>
$b$	Non-employment value	498.5869	70% average wage
Jointly Calibrated Parameters			
$\varphi$	Aggregate shock rate	1.1486	persistence of average productivity = 0.681
$\zeta$	Step size	0.0037	st dev of productivity = 0.0064

Note: <sup>(1)</sup> Hall and Milgrom (2008); <sup>(2)</sup> CPS data; <sup>(3)</sup> Backus, Henriksen, and Storesletten (2008); <sup>(4)</sup> Abowd and Kramarz (2003)

$\lambda = 0.009$ .

We target average features of the US labor market to jointly estimate the parameters  $a$ ,  $y_\ell$ ,  $\mu$ ,  $\Gamma$ ,  $c$ ,  $b$ , and  $\alpha$ . Because of that, we consider the average aggregate productivity value  $z = 1$  and compute the corresponding simulated steady state counterparts of the targets.<sup>14</sup> With regard to the capital elasticity  $a$ , we target the capital to annual output ratio. Backus, Henriksen, and Storesletten (2008) also document that the ratio varies from 2.4 to 2.8 since 1970. We take the intermediate value 2.6 for this target, which becomes 31.2 in monthly terms. Notice that if the steady state equilibrium capital for type- $h$  workers were  $\bar{k}_h$ , then this target would directly yield  $a = 31.2(r + \delta) = 0.2621$ . The simulated capital output ratio is 0.2626.

We set the parameter  $b$  to match the 70 percent of the simulated average wage, which lies within the range from the 0.62 set by Ebell and Haefke (2009), 0.72 implied in Hall and Milgrom (2008) and 0.75 assumed in Costain and Reiter (2008). It encompasses both unemployment benefits and the value of home production and leisure. The calibrated  $b$  to average wage ratio is 0.66. Hall and Milgrom (2008) also assume that the ratio of vacancy cost  $c$  to the simulated average quarterly wage per hire is 14%. Marginally different, we use and perfectly match the 13% ratio reported by Abowd and Kramarz (2003), who collect data from the 1992 French Wage Structure Survey for a representative sample of establishments of at least 20 employees in the manufacturing, construction and service industries.<sup>15</sup>

We target the average monthly job-finding probability. In our dataset, the average predicted probability of becoming employed amounts to 0.2838.<sup>16</sup> Its simulated counterpart is 0.3031. Hall and Milgrom (2008) report an unemployment to vacancy ratio of 2 for the 2000 to 2007 period. Adjusting for temporary layoffs both in the numerator and denominator, the ratio target amounts to 2.4144, whereas the simulated ratio is 2.5767.<sup>17</sup>

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<sup>14</sup>The steady state economy is understood as the  $\varphi = 0$  scenario.

<sup>15</sup>Hagedorn and Manovskii 2008 add a cost of idle capital, which multiplies by a factor of four the management time cost. We do not include such potential idle capital costs in our analysis.

<sup>16</sup>This rate may look too low compared to the values reported in the literature. For example, Shimer (2005b) estimates it at 0.45. The difference is explained by our exclusion of the job-seekers in temporary layoff. Using CPS data since 1994, Fujita and Moscarini (2012) also find comparable numbers for workers who have been permanently laid-off, and around a 0.5 job-finding probability for those in temporary layoff.

<sup>17</sup>Recall that the actual ratio  $u/v = 2$  and  $x = 14.65$  is the percentage of temporary layoff unemployment. Assuming that all temporarily laid off workers are rehired and fill the vacancies of their former employers, the ratio of unemployed not in temporary layoff to the mass of vacancies not filled with a former worker equals  $u(1 - x)/(v - ux) = 2.4144$ .

We have two parameters,  $\mu$  and  $y_\ell$ , related to the dispersion in the economy. Given that  $y_h$  is normalized, targeting an average value fully identifies  $y_\ell$ . To have a sense of the variation and, hence, of  $\mu$ , we target the standard deviation of the log entry wage residuals of a standard Mincerian regression. Such data value and its simulated counterpart amount to 0.3386 and 0.3157, respectively. Finally, we target the average vacancy duration. For the 1985-2001 period, Coles and Petrongolo (2008) estimate that about 40% of newly unemployed workers and of newly posted vacancies registered in UK Jobcenters are on the short side of the market, i.e. they become re-employed and filled in days, respectively. Indeed, they estimate that 30% of the new vacancies are filled on the posting day. This large percentage of vacancies filled in a very short period of time drives the average duration down and is likely to be highly correlated with temporary layoffs. They report that the average duration of a vacancy was slightly over 3 weeks, whereas it jumps to 6 weeks for firms on the long side. van Ours and Ridder (1992) in turn report that vacancy duration ranges from 0.9 to 2.1 months in the Netherlands during the 1980s. Barron, Berger, and Black (1997) document that a US vacancy lasts 13.39, 17.21 and 30.35 days according to the Employment Opportunity Pilot Project 1980 (EOPP), the EOPP 1982 and the Upjohn 1993 surveys, respectively. DeVaro (2005) studies how the vacancy duration depends on the recruitment method and, using data from the Multi-City Study of Urban Inequality survey for 1992 to 1995, documents that the average duration is slightly below 4 weeks. As a compromise, our benchmark calibration targets a 5 weeks expected duration of a vacancy and the simulated duration is 5.3 weeks. **We perform a robustness check with regard to this target and find no significant difference if duration is set at 4 weeks instead.**

Note that our calibration does not explicitly target either the unemployment rate or duration-related variables. Regarding the latter, the obvious explanation is that duration effects are an object of analysis. The unemployment rate of the calibrated steady-state economy is 2.3564%, whereas it raises to 4.44% if temporary layoffs are not excluded from the sample. The reason for this low unemployment rate is because the separation rate falls from 0.15 to 0.12, whereas the job-finding rate only from 0.45 to 0.28 when discarding temporary laid-off workers

Now, we consider the stochastic economy.<sup>18</sup> We deal with  $n = 31$  aggregate productivity values, with a constant distance  $\zeta$  between two consecutive ones. That is,  $z \in \{1 - \frac{n-1}{2}\zeta, \dots, 1, \dots, 1 + \frac{n-1}{2}\zeta\}$ . Conditional on the arrival of an aggregate productivity shock, the transition matrix of the Markov

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<sup>18</sup>We largely use Shimer (2005a) as reference.

process is

$$\pi(z_{i+1}, z_i) = \frac{1 - \frac{z_i - 1}{\frac{n-1}{2}\zeta}}{2}, \quad \pi(z_{i-1}, z_i) = \frac{1 + \frac{z_i - 1}{\frac{n-1}{2}\zeta}}{2}$$

and  $\pi(z_2, z_1) = \pi(z_{n-1}, z_n) = 1$ . Therefore, we have a mean-reverting aggregate productivity process. We simulate data for an exponential variable to record the time spans between aggregate productivity shocks. Then, compute the monthly average productivity in the economy drawing new aggregate productivity values when indicated by the exponential draws. After removing the first 1000 months, we compute the quarterly average of the log monthly productivities for 72 quarters and HP-filter this time series with a smoothing parameter 1600 as usual in the literature. The goal is to calibrate the two parameters  $\varphi$  and  $\zeta$  to match the persistence and standard deviation of the residuals of the AR(1) process of actual log productivities. Therefore, we repeat this exercise 10000 times and compute the averages of the model counterparts of our targets. The actual persistence and standard deviation are 0.7215 and 0.00659, respectively, whereas their (average) simulated counterparts amount to ?? and ??. See the Appendix for further details.

## 5 Unemployment Duration Effects in the Steady State Economy

To analyze the effects of unemployment duration on hazard rates from unemployment and entry wages, we look at the steady state equilibrium. Therefore, in this section we impose that there are no aggregate shocks to the economy,  $\varphi = 0$ . We first state the uniqueness of the steady state equilibrium. Then, we show that both hazard rates and entry wages decline with unemployment duration in equilibrium. Finally, we quantitatively estimate the composition effect on the decline of hazard rates and entry wages from unemployment.

The following proposition states that the steady state equilibrium is unique.

**Proposition 5.1** *There exists a unique steady state equilibrium.*

For notational concreteness, we refer in this section to the tuple  $(\theta_i, k_i, w_i)_i$  as the equilibrium allocation. We can rewrite the unemployment dynamics (4) as

$$u_i = -u_i\nu(\theta_i) + (1 - u_i)\lambda, \quad \text{with } i \in \{\ell, h\} \quad (15)$$

As a result, the steady state unemployment rate of the economy amounts to

$$u = \mu \frac{\lambda}{\lambda + \nu(\theta_h)} + (1 - \mu) \frac{\lambda}{\lambda + \nu(\theta_\ell)} \quad (16)$$

## 5.1 Theoretical Duration Effects

We now move to the equilibrium properties related to unemployment duration. Let the length of an unemployment spell be denoted by  $\tau \in (0, \infty)$ . The average monthly job-finding rate and entry wages at any duration  $\tau$  are determined by

$$\bar{\nu}_\tau = \frac{\mu(1 - u_h)e^{-\tau\nu(\theta_h)}(1 - e^{-\nu(\theta_h)}) + (1 - \mu)(1 - u_\ell)e^{-\tau\nu(\theta_\ell)}(1 - e^{-\nu(\theta_\ell)})}{\mu(1 - u_h)e^{-\tau\nu(\theta_h)} + (1 - \mu)(1 - u_\ell)e^{-\tau\nu(\theta_\ell)}} \quad (17)$$

$$\bar{w}_\tau = \frac{\mu(1 - u_h)e^{-\tau\nu(\theta_h)}(1 - e^{-\nu(\theta_h)})w_h + (1 - \mu)(1 - u_\ell)e^{-\tau\nu(\theta_\ell)}(1 - e^{-\nu(\theta_\ell)})w_\ell}{\mu(1 - u_h)e^{-\tau\nu(\theta_h)}(1 - e^{-\nu(\theta_h)}) + (1 - \mu)(1 - u_\ell)e^{-\tau\nu(\theta_\ell)}(1 - e^{-\nu(\theta_\ell)})} \quad (18)$$

The two expressions are averaging across types and take into account that, first, a worker of type- $i$  has been unemployed for a length  $\tau$  with probability  $e^{-\tau\nu(\theta_i)}$ , and, second, her monthly hazard rate  $1 - e^{-\nu(\theta_i)}$  only depends on her type and the labor market tightness. Notice that we have discarded the event of UEU transitions in a given month for tractability reasons.<sup>19</sup> The UEU transition rate is indeed negligible in the data. The following proposition shows that the average hazard rate declines as duration increases in spite of constant individual rates. This is the case regardless of whether skilled workers leave unemployment relatively sooner or not. The key element underlying the composition effect is that one of the two types is systematically more likely to find a job than the other one. Nevertheless, since the wages newly employed skilled workers obtain are higher in equilibrium, it is central for the result of declining average wages over unemployment duration that type- $h$  job-finding rates are also higher.

**Proposition 5.2** *The average monthly job-finding rates and entry wages decline with unemployment duration.*

## 5.2 Quantitative Duration Effects

Figures 2(a) and 2(b) plot the actual weekly data of monthly hazard rates and entry wages as well as their simulated counterparts according to our benchmark calibration. To compare the actual data with the data simulated from our model, we report the values normalized by the value at unemployment duration of one week. The horizontal axis is at the daily frequency. The monthly job-finding rate declines by 48% after one quarter relative to the first week. The sorting mechanism

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<sup>19</sup>Indeed, assuming that separation rates are constant and type-independent, correcting for this issue would have no effect on the quantitative assessment of the next subsection as we look at normalized values and, hence, such correcting factors would cancel out.

accounts for 29.71% according to our calibration. The model also multiplies by a factor of almost 9 the 2.34% wage fall after the first quarter. This suggests that there is a state dependent component that restrains wages from falling that much, as shown by Fernández-Blanco and Preugschat (2012).

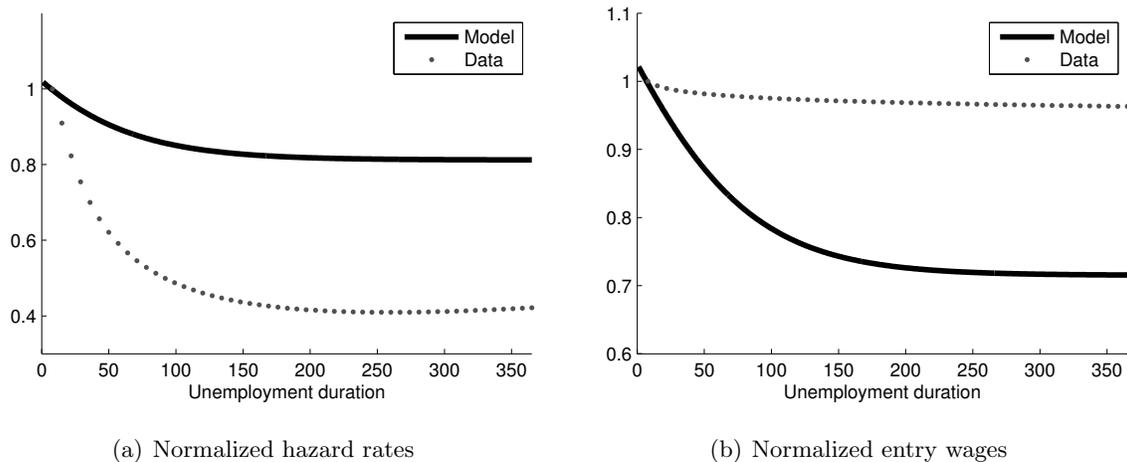


Figure 2: Duration Distributions of Normalized Hazard Rates and Entry Wages

Note: The horizontal axis is measured in days.

## 6 Cyclicity of Labor Market Outcomes

We now analyze the stochastic version of our economy in which aggregate productivity and separation shocks arrive at Poisson rate  $\varphi$ . The main result of this section is that the composition of the unemployment pool barely explains the cyclicity of neither job-finding rates nor entry wages.

Proposition 3.1 implies that the equilibrium is block recursive; hence, the capital levels, wages and search decisions can be determined independently of the unemployment rates and the composition of the unemployment pool. Therefore, the equilibrium variables are jump variables, whereas the state variables adjust over time. This equilibrium feature greatly simplifies the analysis of the stochastic economy.

### 6.1 Productivity Shocks

We start the analysis hitting the economy only with productivity shocks. To get a visible sense of the quantitative importance of the composition component underlying the fluctuations in job-finding rates and entry wages, we first look at a particular time path for these two variables. Once

again, after removing the first 1000 quarterly simulated data, we record the values for the next 72 quarters. Figure 6.1 depicts the dynamics of the quarterly averages of the simulated data, corresponding the right axis to entry wages. Clearly, these two variables co-move in the same direction.

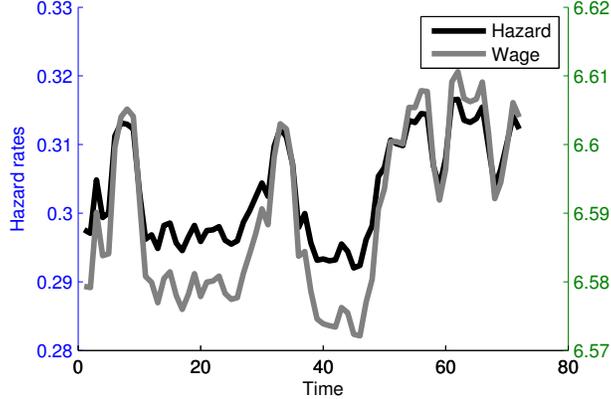


Figure 3: Simulated Hazard Rates and Entry Wages

Note: The values stand for the quarterly average of the correspondent variable. Wages are reported in logs.

Now, we decompose these two variables into their composition and real components following Shimer (2012). To be specific, let  $X(t)$  be a variable generic symbol referring to either hazard rates or entry wages, being  $X_i(t)$  its counterpart restricted to the type  $i$  for a given month  $t$ . Also, let  $m_i(t)$  stand for the mass of type- $i$  workers at month  $t$  who are related to variable  $X$ . That is, if  $X$  stands for hazard rates, then  $m_i(t) = u_i(t)\mu_i$  amounts to the total unemployment of type  $i$  at the beginning of month  $t$ . If  $X$  measures entry wages instead, then  $m_i(t) = \mu_i u_i(t)(1 - e^{-\nu(\theta_i)})$  refers to the mass of newly employed of type  $i$  at the end of month  $t$ .<sup>20</sup> We define the composition and real components of variable  $X$ , respectively, as

$$X^c(t) \equiv \frac{\sum_i m_i(t) \bar{X}_i}{\sum_i m_i(t)}; \quad X^r(t) \equiv \frac{\sum_i \bar{m}_i X_i(t)}{\sum_i \bar{m}_i}, \quad (19)$$

where the averages are defined over time,  $\bar{X}_i \equiv \frac{\sum_t X_i(t)}{T}$  and  $\bar{m}_i \equiv \frac{\sum_t m_i(t)}{T}$ , and  $T = 216$  months. The composition component  $X^c$  is meant to capture the variations of variable  $X$  that stem from the dynamics in the composition of the unemployment pool. The real component  $X^r$  instead disregards any changes in the relative size of the groups and weighted averages the group-specific values of

<sup>20</sup>To be precise, the tightness  $\theta$  may change during the month if a shock hits the economy. In this case, we add up the newly employed across all possible states that take place within a given month.

the variable in question. The quarterly averages of these two components of the simulated time series for hazard rates and wages are plotted in Figures 4(a) and 4(b). The solid lines correspond to the composition time series, whereas the dashed curves stand for the real components. The first graph plots the hazard rates, whereas the second refers to entry wages in logs. It is apparent that all the movements over the cycle in job-finding rates and entry wages stem from the real component, i.e. from variations in the job-finding rates and wages instead of variations in the relative masses of types in the economy. This is the case because, although the high-ability workers amount to approximately 22% of the total labor force, only do they account for less than 3% of total unemployment on average and such a percentage remains very stable over the cycle.

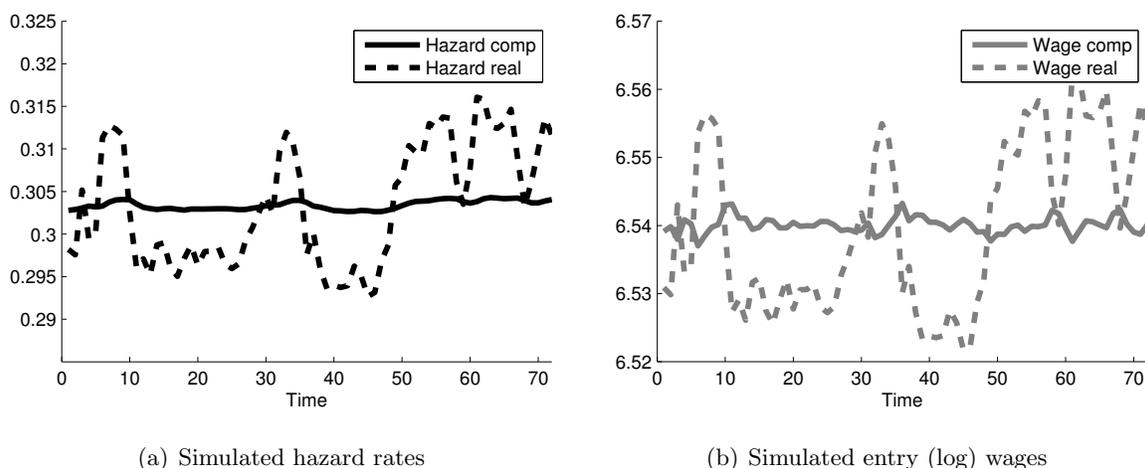


Figure 4: Decomposition of Hazard Rates and Entry Wages

Note: The values stand for the quarterly average of the correspondent variable. Wages are reported in logs.

Although the above figures already give us a sense of the cyclical and volatility of the simulated composition and real components of each variable, Table 2 reports the correlation with average productivity and the standard deviation of each time series. We do so by first HP-filtering 10000 realizations of the time path, computing their respective two statistics and, then, averaging over them. Notice that the elasticity of entry wages and productivity is almost one. This is in line with the findings of Haefke, Sonntag, and van Rens (2012).

The real components of hazard rates and entry wages are highly procyclical and as volatile as job-finding rates and wages, respectively. In contrast, the composition component of wages is highly countercyclical, unlike its counterpart of hazard rates. Likewise, the volatility of the equilibrium and real time series are much larger than their composition counterparts.

Table 2: Cyclicality

	Productivity	$F$	$F^c$	$F^r$	$W$	$W^c$	$W^r$
Correlation with Productivity							
Simulated data	1	0.9971	0.6777	0.9983	0.9838	-0.5719	0.9987
Actual data							
Standard Deviation							
Simulated data		0.0170	0.0008	0.0175	0.0078	0.0012	0.0085
Actual data		0.04047			0.02917		

Note:  $F$  stands for 'Job-finding rates' and  $W$  for 'Entry wages'. All variables stand for the cyclical component computed as the difference between the quarterly averaged logs of the variable and the HP trend with smoothing parameter 1600.

To understand why the composition component of the job-finding rates is procyclical, we can rewrite expression (19) as

$$F^c(t) \equiv \omega_f(t)\bar{F}_h + (1 - \omega_f(t))\bar{F}_\ell = \bar{F}_\ell + \omega_f(t)(\bar{F}_h - \bar{F}_\ell) \quad (20)$$

According to Proposition 3.8,  $\bar{F}_h > \bar{F}_\ell$ . Thus, it follows that the composition component  $F^c$  is procyclical if and only if so is the weight

$$\omega_f(t) \equiv \frac{\mu_h u_h(t)}{\sum_i \mu_i u_i(t)} = \frac{1}{1 + \frac{1-\mu}{\mu} \frac{u_\ell(t)}{u_h(t)}} \quad (21)$$

That is, procyclical of the composition component depends on the mass of type- $\ell$  unemployed workers relative to its skilled counterpart moving in the same direction as aggregate productivity. To put differently, unskilled workers must be relatively more likely to find jobs during booms. Notice here that the separation rate does not depend on the aggregate state of the economy. This is the case because  $u_h$  is very close to its lower bound, whereas  $u_\ell$  is of a higher order of magnitude:  $u_h$  moves around  $10^{-4}$ , whereas  $u_\ell$  does around  $10^{-2}$ .

The counter-cyclical result of the composition component of entry wages is straightforward. Again, Proposition 3.8 implies  $\bar{W}_h > \bar{W}_\ell$ . To make it simple, suppose that there is a shock right at the beginning of the month and no other one during the period. The counterpart of the weight expression (21) is derived as

$$\omega_w(t) \equiv \frac{\mu_h u_h(t)(1 - e^{-\nu(\theta_h)})}{\sum_i \mu_i u_i(t)(1 - e^{-\nu(\theta_i)})} = \frac{1}{1 + \frac{1-\mu}{\mu} \frac{u_\ell(t)(1 - e^{-\nu(\theta_\ell)})}{u_h(t)(1 - e^{-\nu(\theta_h)})}} \quad (22)$$

The ratio  $\frac{u_\ell(t)}{u_h(t)}$  was found to be countercyclical because, we said, unskilled workers are more likely to find jobs as the aggregate productivity increases. That is,  $u_\ell(t)(1 - e^{-\nu(\theta_\ell)})$  is larger than its type- $h$  counterpart. As a result, the weight  $\omega_w$  is countercyclical and so is  $W^c$ .

## 6.2 Productivity and Separation Shocks

[To be added]

## 7 Conclusions

[To be added]

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## 8 Appendix

### 8.1 Surplus and Wage determination.

To determine wages as a Nash-bargaining solution, we better add some details to our setting. A type- $i$  employed worker obtains the flow wage  $w_i(k, \psi)$  and becomes displaced at exogenous rate  $\lambda$ . The aggregate state changes at rate  $\varphi$ . The employment value is defined as

$$rE_i(k, \psi) = w_i(k, \psi) + \lambda(U_i(\psi) - E_i(k, \psi)) + \varphi(\mathbb{E}_\psi(E_i(\bar{k}_i(\psi'), \psi')) - E_i(k, \psi)) \quad (23)$$

The value of a filled vacancy is determined by

$$rJ_i(k, \psi) = p(z, y_i, k) - w_i(k, \psi) - \lambda J_i(k, \psi) + \varphi(\mathbb{E}_\psi(J_i(\bar{k}_i(\psi'), \psi')) - J_i(k, \psi)) \quad (24)$$

The net surplus is defined as  $S_i(k, \psi) \equiv J_i(k, \psi) + E_i(k, \psi) - U_i(\psi)$ . Using above expressions for the value functions, we obtain expression (2).

Upon matching, a firm and a worker set the wage for the instant, and it will be renegotiated inasmuch as a shock hits the economy. Wages are Nash-bargained. Let  $\alpha$  denote the bargaining power of the worker. That is, the equilibrium wage is the solution of the following problem.

$$\max_w (E_i(k, \psi) - U_i(\psi))^\alpha (J_i(k, \psi) - V(\psi))^{1-\alpha} \quad (25)$$

The solution is characterized by  $J_i(k, \psi) = (1 - \alpha)S_i(k, \psi)$ . Using this last equation to replace the value  $J$  in the above Bellman equation (24), we conclude that equilibrium wages are determined as

$$w_i(k, \psi) = p(z, y_i, k) - (r + \lambda + \varphi)(1 - \alpha)S_i(k, \psi) + \varphi(1 - \alpha)\mathbb{E}_\psi (S_i(\bar{k}_i(\psi'), \psi')) \quad (26)$$

That is, workers obtain a proportion  $\alpha$ , and firms the remaining  $1 - \alpha$ , of the surplus of the match in addition to the unemployment value  $(r + \varphi)U_i(\psi)$ .

## 8.2 Proofs

### Proof of Proposition 3.1.

The proof is by contradiction. Suppose there is an equilibrium in which a submarket  $k$  is active when aggregate conditions are  $\psi$  and  $\rho_\ell(k, \psi), \rho_h(k, \psi) > 0$ . Since both types of workers enter submarket  $k$ , we have

$$(r + \varphi)U_i(\psi) = b + \alpha\nu(\Theta(k, \psi))S_i(k, \psi) + \varphi\mathbb{E}_\psi (U_i(\psi')), \quad \forall i \in \{\ell, h\} \quad (27)$$

Firms' profits are defined by expression (3) and average the ex-post profits across workers' types.

Consider now two alternative submarkets  $k'$  and  $k''$ . Instead of  $k$ , they specify capital  $k' \equiv k + \epsilon$  and  $k'' \equiv k - \epsilon$ , respectively, with  $\epsilon$  arbitrarily small, until the arrival of an aggregate shock. By differentiating expressions (27) with respect to  $k$ , we obtain

$$\frac{\partial \theta_i}{\partial k} = \frac{\nu(\Theta(k, \psi))}{\nu'(\Theta(k, \psi))} \frac{-p_k(z, y_i, k)}{S_i(k, \psi)(r + \lambda + \varphi)}, \quad \text{for } i \in \{\ell, h\} \quad (28)$$

where  $\frac{\partial \theta_i}{\partial k}$  measures how much the tightness would change to keep the worker of type  $i$  indifferent between submarkets  $k$  and an alternative one. The above expression takes into account that

$$(r + \lambda + \varphi) \frac{\partial S_i(k, \psi)}{\partial k} = p_k(z, y_i, k) \quad (29)$$

The capital-skill complementarity assumption implies that  $\frac{\partial S_\ell(k, \psi)}{\partial k} < \frac{\partial S_h(k, \psi)}{\partial k}$ . The case study follows:

Case 1: Suppose that  $k \in [\bar{k}_\ell(\psi), \bar{k}_h(\psi)]$ . Then,  $-p_k(z, y_h, k) \leq 0 \leq -p_k(z, y_\ell, k)$ . Therefore,  $\frac{\partial \theta_h}{\partial k} \leq 0 \leq \frac{\partial \theta_\ell}{\partial k}$ , with at least one strict inequality. If  $S_\ell(k, \psi) < S_h(k, \psi)$ , then firms deviating to submarket  $k'$  see a discrete jump in profits as low-ability workers do not apply to  $k'$ . In contrast, if  $S_h(k, \psi) < S_\ell(k, \psi)$ , then type- $h$  workers do not enter submarket  $k''$  and firms deviating to this submarket obtain a discrete jump in profits. Therefore, we reach a contradiction in either case. Therefore, submarket  $k$  cannot be active in equilibrium.

Case 2: Suppose now that  $\bar{k}_h(\psi) < k$ . If  $S_\ell(k, \psi) < S_h(k, \psi)$ , then  $0 < \frac{\partial \theta_h}{\partial k} < \frac{\partial \theta_\ell}{\partial k}$ . Therefore, firms deviating to submarket  $k'$  would obtain a discrete jump in profits. Instead, if  $S_h(k, \psi) < S_\ell(k, \psi)$ , expression (28) does not provide conclusions in the comparison of the partial derivative of the tightness across types, despite being positive. However, notice that if  $\frac{\partial \theta_h}{\partial k} \leq \frac{\partial \theta_\ell}{\partial k}$ , then a deviation to submarket  $k''$  implies a discrete jump in profits by getting rid of high-ability workers. Otherwise, so does a deviation to submarket  $k'$  which only receives applications from type- $\ell$  workers. Once again, we reach a contradiction in either case and, hence, submarket  $k$  cannot be active in equilibrium.

It remains to analyze the subcase  $S_\ell(k, \psi) = S_h(k, \psi)$  in the two cases above. In this case, firms' profits are

$$N(k, \psi) = \eta(\Theta(k, \psi))S_i(k, \psi),$$

and by differentiating with respect to  $k$  and using the above expressions for the partial derivatives of the tightness and the surplus, we obtain

$$\frac{\partial N(k, \psi)}{\partial k} = \eta'(\Theta(k, \psi)) \frac{\partial \Theta(k, \psi)}{\partial k} S_y(k, \psi) + \eta(\Theta(k, \psi)) \frac{\partial S_y(k, \psi)}{\partial k} = \frac{p_k(z, y, k) \nu(\Theta(k, \psi))^2}{r + \lambda + \varphi \nu'(\Theta(k, \psi))},$$

where  $S_y$  is the surplus linked to the type of workers that enter the submarket in question.

If the surpluses are equal to each other, we have  $\frac{\partial \theta_h}{\partial k} < \frac{\partial \theta_\ell}{\partial k}$ . Only do type- $\ell$  workers apply to  $k''$  and  $\frac{\partial N(k, \psi)}{\partial k} < 0$  as  $p_k(z, y_\ell, k) < 0$ . If  $k = \bar{k}_\ell(\psi)$ , then a deviation of firms to submarket  $k'$  obtains larger profits as  $p_k(z, y_h, k) > 0$ . Therefore, we reach a contradiction in either case and, hence, submarket  $k$  cannot be active in equilibrium. ||

**Proof of Lemma 3.2.** By the definition of the surplus value equation (2), we have that function

$S_h - S_\ell$  satisfies the following equation at submarket  $k$ .

$$(r + \lambda + \varphi + \alpha\nu(\Theta(k, \psi))) (S_h - S_\ell)(k, \psi) \geq f(z, y_h, k) - f(z, y_\ell, k) + \varphi \mathbb{E}_\psi (S_h(\bar{k}_h(\psi'), \psi') - S_\ell(\bar{k}_\ell(\psi'), \psi')) \quad (30)$$

The first term of the right side of the inequality is positive because of the capital-skill complementarity. We now show that the second term is also positive. Let  $k_i^*(\psi)$  denote the equilibrium submarket for type- $i$  workers at the state  $\psi$ . Similarly, we can write the surplus gap of ongoing productive pairs as

$$\begin{aligned} S_h(\bar{k}_h(\psi), \psi) - S_\ell(\bar{k}_\ell(\psi), \psi) &= \frac{1}{r+\lambda+\varphi} [p(z, y_h, \bar{k}_h(\psi)) - p(z, y_\ell, \bar{k}_\ell(\psi)) - \\ &\quad - \alpha(\nu(\Theta(k_h^*(\psi), \psi))S_h(k_h^*(\psi), \psi) - \nu(\Theta(k_\ell^*(\psi), \psi))S_\ell(k_\ell^*(\psi), \psi))] + \\ &\quad + \varphi \mathbb{E}_\psi (S_h(\bar{k}_h(\psi'), \psi') - S_\ell(\bar{k}_\ell(\psi'), \psi')) \geq \\ &\geq \frac{1}{r+\lambda+\varphi} [p(z, y_h, \bar{k}_h(\psi)) - p(z, y_\ell, \bar{k}_\ell(\psi)) - \\ &\quad - \alpha \nabla(\psi) (S_h(k_h^*(\psi), \psi) - S_\ell(k_\ell^*(\psi), \psi)) + \\ &\quad + \varphi \mathbb{E}_\psi (S_h(\bar{k}_h(\psi'), \psi') - S_\ell(\bar{k}_\ell(\psi'), \psi'))], \end{aligned} \quad (31)$$

where the inequality comes from defining  $\nabla(\psi)$  as  $\min\{\nu(\Theta(k_h^*(\psi), \psi)), \nu(\Theta(k_\ell^*(\psi), \psi))\}$  if the first argument of the minimum operator is smaller or equal than the second one, and as the positive fraction  $\frac{\nu(\Theta(k_h^*(\psi), \psi))S_h(k_h^*(\psi), \psi) - \nu(\Theta(k_\ell^*(\psi), \psi))S_\ell(k_\ell^*(\psi), \psi)}{S_h(k_h^*(\psi), \psi) - S_\ell(k_\ell^*(\psi), \psi)}$  otherwise. Notice that in the second case the denominator is positive because of the equilibrium zero-profit condition, which implies that  $\eta(\Theta(k_h^*(\psi), \psi))S_h(k_h^*(\psi), \psi) = \eta(\Theta(k_\ell^*(\psi), \psi))S_\ell(k_\ell^*(\psi), \psi)$ .

We now make a guess-and-verify type of argument. We guess the following inequality holds.

$$S_h(\bar{k}_h(\psi), \psi) - S_\ell(\bar{k}_\ell(\psi), \psi) \geq S_h(k_h^*(\psi), \psi) - S_\ell(k_\ell^*(\psi), \psi) \quad (32)$$

This is the case if and only if  $p(z, y_h, \bar{k}_h(\psi)) - p(z, y_h, k_h^*(\psi)) \geq p(z, y_\ell, \bar{k}_\ell(\psi)) - p(z, y_\ell, k_\ell^*(\psi))$ . Proposition 3.3 states that the right side of this last inequality is null and the capital-skill complementarity ensures that the left side is positive. Therefore, the guess is verified. Then, we can claim the surplus gap of inequality (31) satisfies

$$\begin{aligned} S_h(\bar{k}_h(\psi), \psi) - S_\ell(\bar{k}_\ell(\psi), \psi) &\geq \frac{1}{r+\lambda+\varphi+\alpha\nabla(\psi)} [p(z, y_h, \bar{k}_h(\psi)) - p(z, y_\ell, \bar{k}_\ell(\psi)) + \\ &\quad + \varphi \mathbb{E}_\psi (S_h(\bar{k}_h(\psi'), \psi') - S_\ell(\bar{k}_\ell(\psi'), \psi'))] \geq \\ &\geq \sum_{\psi'} (p(z', y_h, \bar{k}_h(\psi')) - p(z', y_\ell, \bar{k}_\ell(\psi'))) \omega(\psi' | \psi_1, \dots, \psi_n), \end{aligned}$$

where the weights  $\omega(\psi'|\psi_1, \dots, \psi_n)$  are positive and so are the terms  $p(z', y_h, \bar{k}_h(\psi')) - p(z', y_\ell, \bar{k}_\ell(\psi'))$  because of capital-skill complementarity. Thus, the surplus gap  $S_h(\bar{k}_h(\psi), \psi) - S_\ell(\bar{k}_\ell(\psi), \psi)$  must be positive at any state  $\psi$ . Therefore, the expected surplus gap term in expression (30) is positive and  $(S_h - S_\ell)(k, \psi) > 0$ .||

**Proof of Proposition 3.3.** We first state and prove the following result.

**Claim 8.1** *Let  $k$  be a submarket such that either  $\rho_\ell(k, \psi) > 0$  or  $\rho_h(k, \psi) > 0$ . Then*

$$\sum_i \rho_i(k, \psi)(1 - \alpha)S_i(k, \psi) \geq (1 - \alpha)S_\ell(k, \psi). \quad (33)$$

**Proof of Claim 8.1.**

Expression (33) is equivalent to

$$(1 - \rho_\ell(k, \psi))S_h(k, \psi) \geq (1 - \rho_\ell(k, \psi))S_\ell(k, \psi). \quad (34)$$

We analyze two cases. If  $\rho_\ell(k, \psi) = 1$ , then, expression (34) holds with equality. Otherwise, if  $\rho_\ell(k, \psi) < 1$  and  $\rho_h(k, \psi) > 0$ , the inequality holds because of Lemma 3.2.||

We first show that for any  $k \in \mathcal{X}_\psi$  such that  $\theta_\ell(\psi) \equiv \Theta(k, \psi) < \infty$  and  $\rho_\ell(k, \psi) > 0$ , the pair  $(\theta_\ell(\psi), S_\ell(k, \psi))$  solves the following program (35). Then, we prove that the solution of the program is uniquely characterized by conditions (5)-(7).

$$\begin{aligned} \max_{\theta \in [0, \infty], S \in [0, \bar{S}_\psi]} \quad & \nu(\theta)S \\ \text{s. to} \quad & \eta(\theta)(1 - \alpha)S \geq c \end{aligned} \quad (35)$$

Proposition 3.1 ensures that if  $k$  is an active submarket such that  $\rho_\ell(k, \psi) > 0$ , then  $\rho_\ell(k, \psi) = 1$ . As a result, the first equilibrium condition implies that the constraint of problem (35) evaluated at the pair  $(\theta_\ell(\psi), S_\ell(\psi))$  holds with equality. We will prove that it is a maximizer by contradiction. Suppose there exists  $(\theta', S') \in [0, \infty] \times [0, \bar{S}_\psi]$  such that  $\nu(\theta')\alpha S' > \nu(\theta_\ell(\psi))\alpha S_\ell(k, \psi)$ , and  $\eta(\theta')(1 - \alpha)S' \geq c$ . The functional equation (2) is a contraction mapping on the set of continuous functions; hence, the solution function is continuous and there must exist  $k'$  such that  $S_\ell(k', \psi) = S'$ . By definition of the equilibrium beliefs,  $\Theta(k', \psi) \leq \theta'$ . Therefore,

$$\eta(\Theta(k', \psi)) \sum_i \rho_i(k', \psi)(1 - \alpha)S_i(k', \psi) \geq \eta(\Theta(k', \psi))(1 - \alpha)S_\ell(k', \psi) > \eta(\theta')(1 - \alpha)S_\ell(k', \psi) \geq c,$$

where the first inequality comes from Claim 8.1. This implies that a deviation to submarket  $k'$  would be profitable contradicting the assumption that  $k$  is an active submarket in equilibrium. Therefore, the pair  $(\theta_\ell, S_\ell(k, \psi))$  is a solution of problem (35).

Now, we benefit from the fact that the constraint must hold with equality to rewrite program (35) as

$$\max_{\theta \in [0, b_\psi]} \theta,$$

where  $\eta(b_\psi)$  equals  $\frac{c}{(1-\alpha)\bar{S}_\psi}$ , and  $\bar{S}_\psi \equiv \max_k S_\ell(k, \psi)$  is the maximum surplus given the current aggregate state  $\psi$ . Thus, the maximizer  $\theta$  is  $b_\psi$  and conditions (5) hold in equilibrium. The surplus  $S_\ell(k, \psi)$  is a strictly concave function of  $k$ ; hence, the maximizing capital level is uniquely determined by the first order condition (7) and equals  $\bar{k}_\ell(\psi)$ .

By using conditions (5) to replace the surplus value in the functional equation (2) and, then, dividing both sides of the resulting expression by  $r + \lambda + \varphi$ , we are left with

$$\frac{1}{\eta(\Theta(k(\psi), \psi))} = \frac{p(z, y_\ell, k(\psi)) - b}{r + \lambda + \varphi} \frac{1 - \alpha}{c} - \alpha \frac{\Theta(k(\psi), \psi)}{r + \lambda + \varphi} + \frac{\varphi}{r + \lambda + \varphi} \mathbb{E}_\psi \left( \frac{1}{\eta(\Theta(k(\psi'), \psi'))} \right)$$

This last functional equation can be thought of as a contraction mapping; hence, it admits a unique solution, which must be the composite function  $f(k, \psi) \equiv 1/\eta(\Theta(k, \psi))$ . By evaluating it at the solutions of equations (7), we obtain the tightness vector  $(\Theta(\bar{k}_\ell(\psi), \psi))_\psi$  as the inverse of function  $1/\eta$  of the resulting values. ||

**Proof of Proposition 3.4.** We first show that condition 1 of problem (P) holds. Consider state  $\psi \in \Psi$ . We show arguing by contradiction that for any active submarket  $k \in \mathcal{X}_\psi$  such that  $\theta \equiv \Theta(k, \psi) < \infty$  and  $\rho_h(k, \psi) > 0$ , the vector  $(\theta, k)$  solves problem  $(P(\psi))$  given the vector  $(U_i(\psi'))_{\psi'}$ .

Once again, Proposition 3.1 ensures that if  $k$  is such that  $\rho_h(k, \psi) > 0$ , then  $\rho_h(k, \psi) = 1$ . As a result, the first equilibrium condition implies that the first constraint of problem  $(P(\psi))$  holds with equality. By definition of the equilibrium beliefs, the second constraint also holds given the equilibrium values  $(U_\ell(\psi))_\psi$ . Therefore, the pair  $(\theta, k)$  belongs to the constraint set. Now, suppose that there exists  $(\theta', k')$  such that  $\nu(\theta')\alpha S_h(k', \psi) > \nu(\theta)\alpha S_h(k, \psi)$ , and satisfies the two constraints of problem  $(P(\psi))$ . Because  $\nu$  is an increasing continuous function, there must exist  $\tilde{\theta} < \theta'$  such that  $\nu(\tilde{\theta})\alpha S_h(k', \psi) = \nu(\theta)\alpha S_h(k, \psi)$ . By definition of the equilibrium beliefs,  $\Theta(k', \psi) \leq \tilde{\theta}$ . It must be the case that type- $\ell$  workers are strictly worse off in submarket  $k'$  because

$b + \nu(\Theta(k', \psi))\alpha S_\ell(k', \psi) < b + \nu(\theta')\alpha S_\ell(k', \psi) \leq (r + \varphi)U_\ell(\psi) - \varphi\mathbb{E}_\psi(U_\ell(\psi))$ . It follows that  $\rho_\ell(k', \psi) = 0$  and  $\Theta(k', \psi) = \tilde{\theta}$ . Therefore,

$$\eta(\Theta(k', \psi)) \sum_i \rho_i(k', \psi)(1 - \alpha)S_i(k', \psi) = \eta(\Theta(k', \psi))(1 - \alpha)S_h(k', \psi) > \eta(\theta')(1 - \alpha)S_h(k', \psi) \geq c$$

This result contradicts the assumption that submarket  $k$  is active in equilibrium because a deviation to submarket  $k'$  would be profitable. Thus, the equilibrium pair  $(\theta, k)$  is a solution of problem  $(P(\psi))$ . Finally, the second condition of problem (P) holds by definition of equilibrium.  $\parallel$

**Proof of Proposition 3.5.** Consider the state  $\psi \in \Psi$  and, given the vectors  $(U_i(\psi'))_{\psi'}$ , the associated problem  $(P(\psi))$ . Due to continuity of the objective function and compactness of the constraint set of the problem, the Weierstrass theorem applies to ensure the existence of a solution. Indeed, the constraint set is non-empty because e.g. the pair zero tightness and a capital level  $k$  being such that the surplus  $S_h(k, \psi)$  is positive satisfies both constraints.

We will show later that the solution always satisfies the first constraint with equality. Then, using the constraint to replace the surplus from the objective function, the program can be rewritten as maximizing the identity function  $\theta$  subject to the two constraints of program  $(P(\psi))$ . Therefore, the maximizer is the maximum feasible tightness value. Hence, for every state  $\psi$ , there exists a unique  $\theta(\psi)$ . Using the first constraint, we obtain a unique capital surplus value  $S(\psi)$ . Since the flow net returns from capital are strictly decreasing in capital in the interval of interest  $[\bar{k}_h(\psi), \infty)$ , the contraction mapping results for the functional equation (2) ensure that the surplus function is also strictly decreasing in capital within that interval; hence, there exists a capital level  $k(\psi)$  whose associated surplus is  $S_h(k(\psi), \psi) = S(\psi)$ . Moreover, given the obtained vectors  $(\theta(\psi))_\psi$  and  $(S_h(k(\psi), \psi))_\psi$ , the unemployment values  $(\tilde{U}_h(\psi))_\psi$  obtain as a solution of the functional equation (1).

Now, let  $H$  be a function defined on  $\mathbb{R}^{|\Psi|}$  as  $H((U_h(\psi))_\psi) = (\tilde{U}_h(\psi))_\psi$ . To show the existence of solution of problem  $(P)$ , we just need to show existence of a fixed point of  $H$ . Notice that function  $H$  is defined over the compact set  $[0, \bar{U}]^{|\Psi|}$ , with  $\bar{U}$  sufficiently large. Furthermore, its continuity is inherited from the continuity of the functions involved in the maximization problem. Therefore, the Brouwer's fixed-point theorem applies.

Now, to characterize the solution of the program  $(P(\psi))$  given state  $\psi$ , we define the Lagrangian

as

$$\begin{aligned} \mathcal{L} = & \nu(\theta)\alpha S_h(k, \psi) + \xi_1 (\eta(\theta)(1 - \alpha)S_h(k, \psi) - c) - \\ & -\xi_2 (b + \nu(\theta)\alpha S_\ell(k, \psi) + \varphi\mathbb{E}_\psi (U_\ell(\psi)) - (r + \varphi)U_\ell(\psi)) \end{aligned}$$

where  $\xi_1$  and  $\xi_2$  stand for the non-negative multipliers associated to the first and second constraints, respectively.<sup>21</sup> The Kuhn Tucker (necessary) conditions are

$$\nu'(\theta)\alpha (S_h(k, \psi) - \xi_2 S_\ell(k, \psi)) + \xi_1 \eta'(\theta)(1 - \alpha)S_h(k, \psi) \leq 0, \theta \geq 0 \quad (36)$$

$$\text{and} \quad \theta \nu'(\theta)\alpha (S_h(k, \psi) - \xi_2 S_\ell(k, \psi)) + \theta \xi_1 \eta'(\theta)(1 - \alpha)S_h(k, \psi) = 0.$$

$$\frac{\partial S_h(k, \psi)}{\partial k} (\nu(\theta)\alpha + \xi_1 \eta(\theta)(1 - \alpha)) - \xi_2 \nu(\theta)\alpha \frac{\partial S_\ell(k, \psi)}{\partial k} \leq 0, k \geq 0 \quad (37)$$

$$\text{and} \quad k \frac{\partial S_h(k, \psi)}{\partial k} (\nu(\theta)\alpha + \xi_1 \eta(\theta)(1 - \alpha)) - k \xi_2 \nu(\theta)\alpha \frac{\partial S_\ell(k, \psi)}{\partial k} = 0.$$

$$\eta(\theta)(1 - \alpha)S_h(k, \psi) \geq c, \xi_1 \geq 0 \text{ and } \xi_1 [\eta(\theta)(1 - \alpha)S_h(k, \psi) - c] = 0. \quad (38)$$

$$b + \nu(\theta)\alpha S_\ell(k, \psi) + \varphi\mathbb{E}_\psi (U_\ell(\psi)) \leq (r + \varphi)U_\ell(\psi), \xi_2 \geq 0 \quad (39)$$

$$\text{and} \quad \xi_2 [b + \nu(\theta)\alpha S_\ell(k, \psi) + \varphi\mathbb{E}_\psi (U_\ell(\psi)) - (r + \varphi)U_\ell(\psi)] = 0.$$

This set of complementary slackness conditions delivers the characterization of the solution of the problem. Let us analyze two different cases depending on whether the second constraint is binding or not.

Case 1.  $\xi_2 = 0$  Condition (37) implies that  $k > 0$  as  $\lim_{k \rightarrow 0} \frac{\partial S_h(k, \psi)}{\partial k} = \infty$ . This same complementary slackness condition leads to the necessary condition (12),  $p_k(z, y_h, k) = 0$ , which has a unique solution due to the assumptions on the production technology. Notice that if the first constraint were slack, then  $\xi_1 = 0$  because of condition (38). As a result,  $\theta = \infty$  would follow from the complementary slackness condition (36), contradicting the same first constraint. Thus, the necessary condition (9) holds. However, the second constraint must hold for the pair  $(\theta, x)$  obtained above to be the solution of the problem. The second case needs to be analyzed only if this second constraint fails to hold.

Case 2.  $\xi_2 > 0$  Condition (13) must hold according to the complementary slackness condition (39). We now turn to show by contradiction that the first constraint cannot be slack either in this case. Suppose that  $\xi_1 = 0$  and  $\eta(\theta)(1 - \alpha)S_h(k, \psi) > c$ . If  $\theta = 0$ , then condition (39) fails to hold; hence,  $\theta > 0$ . According to condition (36), either  $\nu'(\theta) = 0$  or  $S_h(k, \psi) = \xi_2 S_\ell(k, \psi)$ . The former implies  $\theta = \infty$  and hence a zero job-filling rate for firms making the expected discounted profits null,

<sup>21</sup>To save on notation, we avoid to make the objects  $k, \theta, \xi_j$  dependent on the state  $\psi$ .

which contradicts condition (38). As a result, it must be the case that  $S_h(k, \psi) = \xi_2 S_\ell(k, \psi)$ . In addition, if  $\theta \in (0, \infty)$ , then  $\xi_2 = 1$  according to condition (37) since  $\frac{\partial S_h(k, \psi)}{\partial k} > \frac{\partial S_\ell(k, \psi)}{\partial k}$  according to expression (29). Putting these two conditions together, we obtain that  $S_h(k, \psi) = S_\ell(k, \psi)$ . Proceeding along the same lines as in the proof of Lemma 3.2, we know that this fails to be true due to the complementarity between capital and ability. Moreover, condition (??) follows from the complementary slackness condition (37) as  $\frac{\partial S_\ell(k, \psi)}{\partial k} < 0$ . It implies that the solution is such that  $k(\psi) > \bar{k}_h(\psi)$ .

In both cases, we can replace the surplus value in the surplus equation (2) using the zero-profit condition to obtain equation (10).||

**Proof of Proposition 3.6.** An equilibrium is a tuple  $\{G_\psi, \mathcal{X}_\psi, \Theta(\cdot, \psi), \{U_i(\psi), S_i(\cdot, \psi), \rho_i(\cdot, \psi)\}_i\}_\psi$  that satisfies the equilibrium definition 1. Apart from the equilibrium objects already defined in the proposition statement, it remains to determine the surplus function, the off-the-equilibrium beliefs, and the continuum of distributions  $(G_\psi)_\psi$ . Given the vector  $(U_i(\psi'))_{\psi'}$ , the surplus function  $S_i(\cdot, \psi)$  is uniquely defined as the solution of the contraction mapping (2) at any state  $\psi$ . The remaining equilibrium objects are set consistently with the equilibrium definition. In particular, if  $\psi$  is the aggregate state of the economy and  $\mathcal{X}_\psi = \{k_\ell(\psi), x_h(\psi)\}$ , then

$$dG_\psi(k) \equiv \begin{cases} (1 - \mu)u_\ell \Theta(k, \psi), & \text{if } k = k_\ell(\psi) \\ \mu u_h \Theta(k, \psi), & \text{if } k = k_h(\psi) \\ 0, & \text{otherwise.} \end{cases}$$

The construction of distribution  $G$  trivially ensures the equilibrium market-clearing condition. We define the beliefs off-the-equilibrium for all submarket  $k$  as

$$\Theta(k, \psi) \equiv \min_i \hat{\theta}_i(k, \psi),$$

$$\rho_\ell(k, \psi) = \begin{cases} 1, & \text{if } \hat{\theta}_\ell(k, \psi) < \hat{\theta}_h(k, \psi) \\ 0, & \text{otherwise} \end{cases}, \text{ and } \rho_h(k, \psi) = \begin{cases} 1, & \text{if } \hat{\theta}_h(k, \psi) \leq \hat{\theta}_\ell(k, \psi) \\ 0, & \text{otherwise,} \end{cases}$$

where  $\hat{\theta}_i(k, \psi) \equiv \begin{cases} \theta \text{ such that } b + \nu(\theta)\alpha S_i(k, \psi) + \varphi \mathbb{E}_\psi(U_i(\psi')) = (r + \varphi)U_i(\psi), & \text{if } S_i(k, \psi) > 0 \\ \infty, & \text{otherwise} \end{cases}$

Notice that the beliefs are well defined given the assumptions on the matching function  $\nu$  and there is no submarket  $k$  such that both types of workers enter simultaneously. Now, we first show that

firms maximize profits and those are zero in equilibrium for all active submarkets. Second, we prove that workers optimally search for jobs. Again, let  $\psi$  be the current state of the economy. The free entry condition holds in each active submarket because of Propositions 3.3 and 3.5. We turn to show by contradiction that firms maximize profits by entering either active submarket in  $\mathcal{X}_\psi = \{k_\ell(\psi), k_h(\psi)\}$ . We distinguish two cases.

Case 1. Suppose there exists a submarket  $k'$  such that  $\Theta(k', \psi) < \infty$ ,  $\rho_\ell(k', \psi) = 1$  and  $\eta(\Theta(k', \psi))(1 - \alpha)S_\ell(k', \psi) > c$ . To attract low-ability workers, it must be the case that  $b + \nu(\Theta(k', \psi))\alpha S_\ell(k', \psi) + \varphi \mathbb{E}_\psi(U_\ell(\psi)) = (r + \varphi)U_\ell(\psi)$ . Because of continuity and monotonicity of function  $\eta$ , there exists  $\theta' > \Theta(k', \psi)$  such that  $\eta(\theta')(1 - \alpha)S_\ell(k', \psi) = c$ . Since  $\nu$  is an increasing function of the tightness, we have

$$\nu(\Theta(k', \psi))\alpha S_\ell(k', \psi) = (r + \varphi)U_\ell(\psi) - b - \varphi \mathbb{E}_\psi(U_\ell(\psi)) < \nu(\theta')\alpha S_\ell(k', \psi),$$

which contradicts the assumption that  $(\theta_\ell(\psi), k_\ell(\psi))$  is the maximizer of problem (8) and  $(r + \varphi)U_\ell - b - \varphi \mathbb{E}_\psi(U_\ell(\psi))$  is the maximum value. Therefore, there is no profitable deviation that attracts only low-ability workers.

Case 2. Suppose there exists a submarket  $k'$  such that  $\Theta(k', \psi) < \infty$ ,  $\rho_h(k', \psi) = 1$  and  $\eta(\Theta(k', \psi))(1 - \alpha)S_h(k', \psi) > c$ . Again, high-ability workers enter the submarket if  $b + \nu(\Theta(k', \psi))\alpha S_h(k', \psi) + \varphi \mathbb{E}_\psi(U_h(\psi)) = (r + \varphi)U_h(\psi)$ . Further, the following inequality follows from the off-the-equilibrium beliefs,  $b + \nu(\Theta(k', \psi))\alpha S_\ell(k', \psi) + \varphi \mathbb{E}_\psi(U_\ell(\psi)) \leq (r + \varphi)U_\ell(\psi)$ . We distinguish two subcases. First, if this last inequality is strict, then there must exist  $\theta' > \Theta(k', \psi)$  such that the two constraints of problem  $(P(\psi))$  hold at  $(\theta', k')$ . Because of the properties of the matching technology, we obtain that  $\nu(\theta')(1 - \alpha)S_h(k', \psi) > (r + \varphi)U_h(\psi) - b - \varphi \mathbb{E}_\psi(U_h(\psi))$ , which contradicts our assumption that  $(\theta_h(\psi), k_h(\psi))$  is the maximizer. Second, if the inequality is indeed an equality, then  $\frac{\partial \theta_\ell(k', \psi)}{\partial k'} > \frac{\partial \theta_h(k', \psi)}{\partial k'}$ , where  $k'$  is the capital level at state  $\psi$  of  $k'$ , as shown in the proof of Proposition 3.1. This means that there exists a submarket  $k''$ , which differs from  $k'$  by an arbitrarily small amount, such that the first constraint of problem  $(P(\psi))$  holds, the second one holds with strict inequality and  $b + \nu(\Theta(k'', \psi))(1 - \alpha)S_h(k'', \psi) + \varphi \mathbb{E}_\psi(U_h(\psi)) = (r + \varphi)U_h(\psi)$ . This leads us back to the first case.

It remains to show the equilibrium condition of workers' optimally search. This is the case by construction of the beliefs  $\Theta(\cdot, \psi)$ .

**Proof of Proposition 3.8.** Conditions (5)-(7) and (10)-(13) characterize the equilibrium out-

come. Let  $\psi$  be the state of the economy. Let us define  $\bar{k}_y(\psi)$  as the value of  $k$  that satisfies  $p_k(z, y, k) = 0$ , given parameter  $y$ . By differentiating with respect to  $y$ , we obtain that  $\frac{\partial \bar{k}_y(\psi)}{\partial y} = -\frac{p_{ky}(z, y, \bar{k}_y(\psi))}{p_{kk}(z, y, \bar{k}_y(\psi))} > 0$ . Because of the equilibrium conditions (7) and (12), we have  $k_\ell(\psi) < k_h(\psi)$ . If the capital level were determined by (13) instead, then we already stated that  $k_\ell(\psi) < \bar{k}_h(\psi) \leq k_h(\psi)$ .

From the zero-profit conditions (5) and (9), we have

$$\eta(\theta_\ell(\psi))S_\ell(k_\ell(\psi), \psi) = \eta(\theta(\psi))S_h(k_h(\psi), \psi). \quad (40)$$

Along similar lines as in the proof of Lemma 3.2, we can show that

$$\nu(\theta_\ell(\psi))S_\ell(k_\ell(\psi), \psi) < \nu(\theta(\psi))S_h(k_h(\psi), \psi).$$

Putting these two expressions together, it follows that  $\theta_\ell(\psi) < \theta_h(\psi)$ . The tightness gap together with expression (40) implies that  $S_\ell(k_\ell(\psi), \psi) < S_h(k_\ell(\psi), \psi)$ . As a result, by the definition of the unemployment value, we also obtain  $U_\ell(\psi) < U_h(\psi)$ . Finally, we can rewrite the entry wage expressions as

$$w_\ell(k_\ell(\psi), \psi) = \alpha p(z, y_\ell, k_\ell(\psi)) + (1 - \alpha)((r + \varphi)U_\ell(\psi) - \varphi \mathbb{E}_\psi(U_\ell(\psi')))$$

$$w_h(k_h(\psi), \psi) = \alpha p(z, y_h, k_h(\psi)) + (1 - \alpha)((r + \varphi)U_h(\psi) - \varphi \mathbb{E}_\psi(U_h(\psi')))$$

Because both terms on the right side are larger for high-ability workers, it follows that  $w_\ell(k_\ell(\psi), \psi) < w_h(k_h(\psi), \psi)$ .||

**Proof of Proposition 5.2** To show that hazard rates and entry wages decline with duration  $\tau$ , it suffices to rewrite the expressions (17) and (18) as

$$\begin{aligned} \bar{v}_\tau &= \frac{\mu\nu(q_h) + (1 - \mu)e^{-\tau(\nu(q_\ell) - \nu(q_h))}\nu(q_\ell)}{\mu + (1 - \mu)e^{-\tau(\nu(q_\ell) - \nu(q_h))}} = \nu(q_\ell) + \frac{\mu(\nu(q_h) - \nu(q_\ell))}{\mu + (1 - \mu)e^{-\tau(\nu(q_\ell) - \nu(q_h))}} \\ \bar{w}_\tau &= \frac{\mu\nu(q_h)w_h + (1 - \mu)e^{-\tau(\nu(q_\ell) - \nu(q_h))}\nu(q_\ell)w_\ell}{\mu\nu(q_h) + (1 - \mu)e^{-\tau(\nu(q_\ell) - \nu(q_h))}\nu(q_\ell)} = w_\ell + \frac{\mu\nu(q_h)(w_h - w_\ell)}{\mu\nu(q_h) + (1 - \mu)e^{-\tau(\nu(q_\ell) - \nu(q_h))}\nu(q_\ell)} \end{aligned}$$

The first expression is decreasing in  $\tau$  regardless of the sign of the term  $\nu(q_h) - \nu(q_\ell)$ . The derivative of the average wage with respect to  $\tau$  is also negative as  $\nu(q_h) > \nu(q_\ell)$  and  $w_h > w_\ell$  according to Proposition 3.8.||

### 8.3 Appendix Calibration

Let  $N_i(t + \Delta)$  denote the mass of newly employed workers of type  $i$  between instant  $t$  and  $t + \Delta$  conditional on being unemployed at instant  $t$ . In the time period  $\Delta$ , we have

$$\frac{N_i(t + \Delta) - N_i(t)}{\Delta} = \frac{1 - \exp -\nu_i \Delta}{\Delta} u_i(t) \quad (41)$$

This is the case because  $N_i(t) = 0$ . Taking limits on both sides of this expression, we obtain  $\dot{N}_i(t) = \nu_i u_i(t)$  and integrating it follows

$$N_i(t) = \quad (42)$$